Conformity assessment of a sample of items – an extension of the JCGM 106:2012

F. Pennecchi[1], I. Kuselman[2],

[1] Istituto Nazionale di Ricerca Metrologica (INRIM), f.pennecchi@inrim.it, Torino, Italy
[2] Independent Consultant on Metrology, ilya.kuselman@bezeqint.net, Modiin, Israel
E-mail (corresponding author): f.pennecchi@inrim.it

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The JCGM 106:2012 document [1] provides guidelines on how to perform conformity assessment (CA) of a scalar property of interest of a single item (a product, material, object, etc.) based on a Bayesian approach. This property is the measurand Y, i.e., the quantity undergoing CA. Modelling the information on Y available ahead of the measurement by a prior probability density function (PDF) $g_0(\eta)$, and describing the measurement process by a likelihood function $h(\eta_m|\eta)$ (η and η_m are the true and measured values, respectively), the post-measurement state of knowledge on Y is derived as the posterior PDF $g(\eta|\eta_m)$:

$$g(\eta|\eta_m) = C\, g_0(\eta)\, h(\eta_m|\eta), \qquad (1)$$

where C is a normalizing constant.
The aim of a CA exercise is to assess the conformance of an η value to a prescribed Tolerance Interval (TI) by checking whether a corresponding measured value η_m, inevitably affected by uncertainty, falls into a desired Acceptance Interval (AI).
The JCGM 106:2012 gives indications on how to calculate specific and global risks of erroneous decisions for both the consumer (c) and the producer (p):

- specific risks (for a specific item)

$$R_c^* = \int_{TI'} g(\eta|\eta_m)\, d\eta \qquad (\eta_m \text{ in AI}), \qquad (2)$$
$$R_p^* = \int_{TI} g(\eta|\eta_m)\, d\eta \qquad (\eta_m \text{ in AI'}), \qquad (3)$$

- global risks (for an item chosen at random from the population)

$$R_c = \int_{TI'} \int_{AI} g_0(\eta)\, h(\eta_m|\eta)\, d\eta_m\, d\eta, \qquad (4)$$
$$R_p = \int_{TI} \int_{AI'} g_0(\eta)\, h(\eta_m|\eta)\, d\eta_m\, d\eta, \qquad (5)$$

where TI' and AI' indicate true and measured values lying outside TI and AI, respectively.
This clear and sounded approach deserves to be extended to more general decision problems. The CA of multicomponent materials or objects, for example, is not reducible to a risk calculation made component by component: multivariate approaches [2] and *total* risks for multicomponent objects were proposed [3, 4]. A further direction toward which the JCGM 106 framework needs to be extended is the CA of a finite sample of N items drawn from a common population. A relevant research activity is currently under development [5, 6], specifically aimed at inspection of lots. In those works, the general idea is to consider the target variable Y as the (true) proportion of nonconforming items in the lot and to work with an appropriate prior PDF for it, together with a suitable likelihood function for the number of items assessed as nonconforming in the lot. Specific and global risks are then calculated, broadening the JCGM 106 scope to a lot inspection.
The work presented here, instead, relies on an alternative approach. It proposes to:

- Calculate specific and global risks for the items in a lot according to the JCGM 106, i.e., by eqs. (2-5); and then to

- Model the probability of the number of items truly conforming within the lot by means of an appropriate probability mass function (PMF).

Specifically, the following two kinds of modelling can be performed:

1) Given a sample of N items, each characterized by specific risks as in eqs. (2) and (3), and knowing that the measured values for K of them are within their AI, the discrete random variable (r.v.) V counting how many of the measured values actually come from corresponding good true values is the sum of N independent Bernoulli r.v. with different success probabilities, that is:

   $V \sim$ Poisson binomial$(1 - R^*_{1c}, \ldots, 1 - R^*_{Kc}, R^*_{(K+1)p}, \ldots, R^*_{Np})$;

2) Considering to randomly draw a sample of N items from the whole population, already characterized by global risks as in eqs. (4) and (5), the discrete r.v. W counting how many false positives, false negatives, true positives and true negatives are in the sample is:

   $W \sim$ multinomial$(N; R_c, R_p, p_{TP}, p_{TN})$ (with $R_c + R_p + p_{TP} + p_{TN} = 1$), where

   $p_{TP} = \int_{TI} \int_{AI} g_0(\eta)\, h(\eta_m|\eta)\, d\eta_m\, d\eta$, (11)

   $p_{TN} = \int_{TI'} \int_{AI'} g_0(\eta)\, h(\eta_m|\eta)\, d\eta_m\, d\eta$, (12)

   are the probabilities of true positives and true negatives, respectively.

Therefore, after applying the standard JCGM 106 approach, the two PMFs mentioned above allow answering questions like: which is the probability that at least a certain number of true values were actually conforming in a sample where K items were within their AI? Or, the other way round, which is the maximum number of conforming true values that can be expected to occur with a desired probability? Moreover, for given values of the global risks, which thresholds can be set for false positives, false negatives and true negatives in order to get a desired quality for a future sample drawn from that population? How would this quality be impacted if the sample size and/or the measurement uncertainty (hence the risk values) changed?

Application examples of the proposed modelling will be shown at the Conference.

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