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*Original*
Risk of a false decision on conformity of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants / Pennecchi, Francesca R; Kuselman, Ilya; da Silva, Ricardo J N B; Hibbert, D Brynn. - In: CHEMOSPHERE. - ISSN 0045-6535. - 202:(2018), pp. 165-176. [10.1016/j.chemosphere.2018.03.054]

*Publisher:*
Elsevier

*Published*
DOI:10.1016/j.chemosphere.2018.03.054

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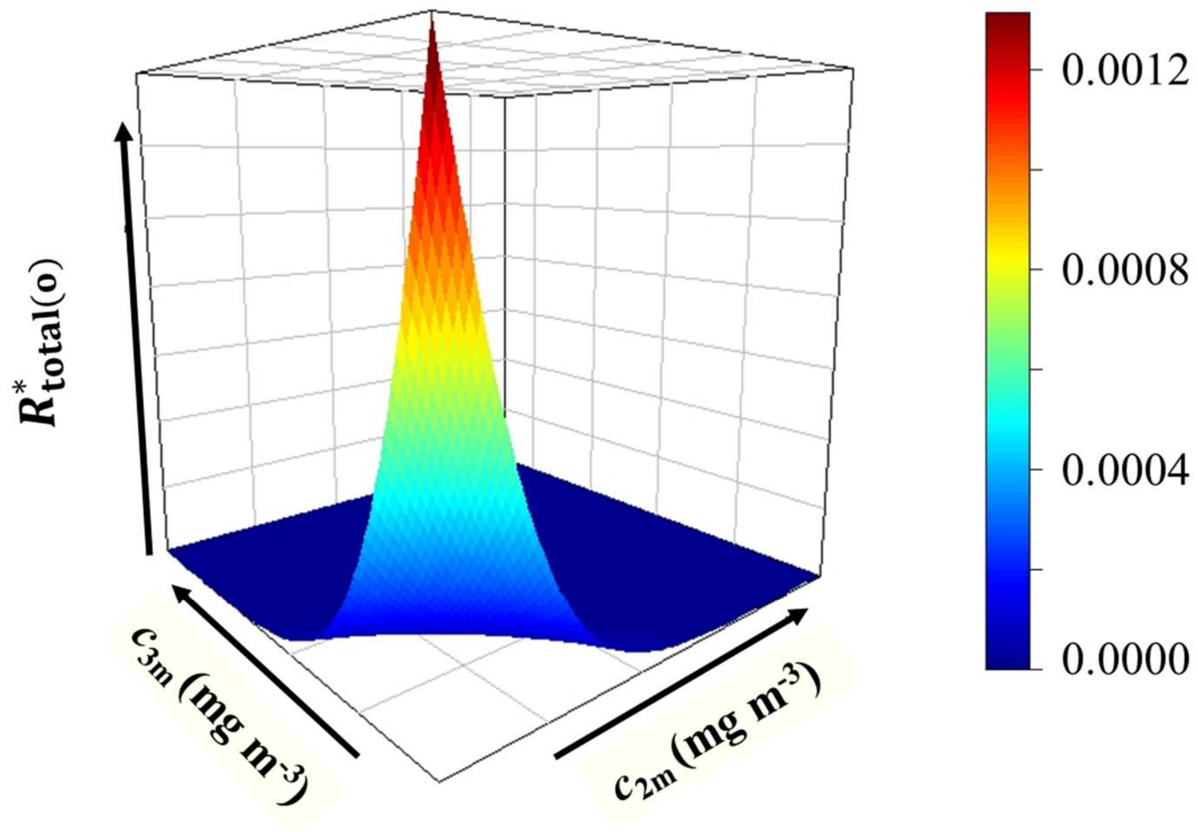
PII: S0045-6535(18)30468-5
DOI: 10.1016/j.chemosphere.2018.03.054
Reference: CHEM 20998

To appear in: *ECSN*

Received Date: 17 November 2017
Revised Date: 6 March 2018
Accepted Date: 7 March 2018

Please cite this article as: Pennecchi, F.R., Kuselman, I., da Silva, R.J.N.B., Hibbert, D.B., Risk of a false decision on conformity of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants, *Chemosphere* (2018), doi: 10.1016/j.chemosphere.2018.03.054.



Dependence of the total risk of overestimation $R^*_{\mathrm{total(o)}}$ of suspended particulate matter concentration in ambient air on the measurement results $c_{i\mathrm{m}}$ in proximity to the three quarries ($c_{1\mathrm{m}}$= 0.250 mg m$^{-3}$; $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ are varying from 0.210 to 0.300 mg m$^{-3}$).

# Risk of a false decision on conformity of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants

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## 1. Introduction

Actual (‘true’) concentration $c_i$ of the $i$-th pollutant, $i$ = 1, 2, …, $n$, in an environmental compartment, e.g. ambient air (Duursma and Carroll (1996); TIMBRE project, Online), should not exceed a regulation or legal tolerance upper limit $T_{\mathrm{U}i}$. ‘Concentration’ is used here as a generic term (Cvitaš, 1996; Tolhurst, 2005; Fuentes-Arderiu, 2013). Comparing a chemical analytical test/measurement result $c_{i\mathrm{m}}$ of the $i$-th pollutant concentration with the $T_{\mathrm{U}i}$ value, one should decide whether the compartment conforms to the regulation or not. Since any result $c_{i\mathrm{m}}$ has an associated measurement uncertainty (Ellison and Williams, 2012; Magnusson et al., 2012), several kinds of risk of a false decision on conformity of the compartment may arise.

The probability of a decision that the actual pollutant concentration does not exceed the limit since $c_{i\mathrm{m}} \leq T_{\mathrm{U}i}$, when it is not correct (i.e. $c_i > T_{\mathrm{U}i}$), is named ‘consumer’s risk’. The ‘consumer’ in the present paper is a habitant whose quality of life (including health) depends on adequate control of the pollutant. Thus, the consumer’s risk is the probability of underestimation of $c_i$ due to measurement uncertainty associated with $c_{i\mathrm{m}}$.

On the other hand, the probability of falsely rejecting the decision on conformity of the compartment to the regulation (i.e. $c_{i\mathrm{m}} > T_{\mathrm{U}i}$ when $c_i \leq T_{\mathrm{U}i}$) is the ‘producer’s risk’. The ‘producer’ here is a plant or another organization – a source of the environment pollution, obliged to pay a fine and/or to invest money for an unnecessary reduction of the pollutant concentration in the case of false nonconformity. The producer’s risk is therefore the probability of overestimation of $c_i$ due to measurement uncertainty in $c_{i\mathrm{m}}$.

For a specified compartment, e.g. ambient air in a certain location at a certain time, such risks are referred to as the ‘specific consumer’s risk’ of underestimation $R^*_{ci(\mathrm{u})}$ and the ‘specific producer's risk' of overestimation $R^{*}_{ci(\mathrm{o})}$ for *i*-th particular pollutant concentration. The risks of incorrect conformity assessment of a compartment randomly drawn from a statistical population of such compartments are the 'global consumer's risk' of underestimation $R_{ci(\mathrm{u})}$ and the 'global producer's risk' of overestimation $R_{ci(\mathrm{o})}$, respectively, as they characterize the environmental quality globally. Evaluation of the particular risks (both specific and global) is described in the JCGM 106 (2012) based on a Bayesian approach to conformity assessment.

However, when concentrations of two or more pollutants are controlled, pollutant-by-pollutant evaluation of the risks is not complete in general, as it does not give an answer to the question of the probability of a false decision on the overall compartment conformity. If conformity assessment for each *i*-th pollutant concentration of a compartment is successful, i.e. the particular specific $R^{*}_{ci}$ or global $R_{ci}$ risks of both under- and overestimation are small enough, the total probability of a false decision concerning conformity of the compartment as a whole (the *total* specific $R^{*}_{\mathrm{total}}$ or *total* global $R_{\mathrm{total}}$ risk) might still be significant.

Fig. 1

A scheme summarizing the used terminology is shown in Fig. 1, where the particular risks described in the JCGM 106 (2012) are shown at the top of the scheme. The *total risk evaluation*, as the task of the IUPAC Project (2016), is highlighted by an ellipse at the bottom of the scheme.

Using the law of total probability for the case of independent quantities (pollutant concentration values and corresponding measurement results) the total risk of underestimation can be evaluated as a combination of the particular risks (Kuselman et al., 2017a). For example, for three pollutions $i$ = 1, 2, 3, assuming independent actual values of each pollutant concentration $c_i$ and independent corresponding measurement results $c_{i\mathrm{m}}$, the total specific risk of underestimation is:

$$R^*_{\text{total(u)}} = R^*_{c1(\text{u})} + R^*_{c2(\text{u})} + R^*_{c3(\text{u})} - R^*_{c1(\text{u})}R^*_{c2(\text{u})} - R^*_{c1(\text{u})}R^*_{c3(\text{u})} - R^*_{c2(\text{u})}R^*_{c3(\text{u})} + R^*_{c1(\text{u})}R^*_{c2(\text{u})}R^*_{c3(\text{u})}. \qquad (1)$$

E.g., for all the particular specific risks $R^*_{ci(\text{u})} = 0.05$, the total specific risk by formula (1) is $R^*_{\text{total}} = 0.14$. Total global risk of underestimation for the three pollutants is:

$$R_{\text{total(u)}} = P(\text{C}_2)P(\text{C}_3)R_{c1(\text{u})} + P(\text{C}_1)P(\text{C}_3)R_{c2(\text{u})} + P(\text{C}_1)P(\text{C}_2)R_{c3(\text{u})} - P(\text{C}_3)R_{c1(\text{u})}R_{c2(\text{u})} - P(\text{C}_2)R_{c1(\text{u})}R_{c3(\text{u})} - P(\text{C}_1)R_{c2(\text{u})}R_{c3(\text{u})} + R_{c1(\text{u})}R_{c2(\text{u})}R_{c3(\text{u})}, \qquad (2)$$

where $P(\text{C}_i)$ is the probability that a measurement result $c_{im}$ is acceptable, i.e. $c_{im} \leq T_{\text{U}i}$. For example, for the particular risks $R_{ci}$ = 0.05 and probabilities $P(\text{C}_i)$ = 0.90 for all $i$, formula (2) gives $R_{\text{total}} = 0.12$.

General expressions for evaluating the total risk of underestimation for any number $n$ of the material components (or pollutants of an environmental compartment) are also provided in the mentioned above reference. Treatment of correlated measurement results for total risk evaluation is discussed in the paper by Kuselman et al. (2017b).

In the present paper, the total risk of overestimation (producer's risk) is formulated in the same Bayesian framework for uncorrelated test results as it was applied in the previous work (Kuselman et al., 2017a) for underestimation (consumer's risk). Core code developed in R programming environment (the R project, Online) for corresponding calculations is also provided. As a case study, total risk values are calculated for conformity assessment of concentration of total suspended particulate matter (TSPM) in ambient air from three independent stone quarries in Israel. In this study TSPM contributed by the *i*-th quarry, $i = 1, 2, 3$, is considered as the *i*-th pollutant. While particular risk values of false decisions on conformity of the *i*-th TSPM concentration, evaluated earlier (Kuselman et al., 2012a), were related to each *i*-th pollutant (*i*-th quarry) separately, the total risk values discussed below allow characterization of conformity of the TSPM concentration in the region of the quarries as a whole. That is important as for the Regulator (the Ministry of Environmental Protection, Online) protecting the inhabitants' quality of life in the area surrounding the quarries, as for the Manufacturers Association (Online) acting in the interests of the stone producers in the country.

## 2. Methods

### 2.1. Raw data

*2.1.1. Test method and likelihood functions*

A measured TSPM concentration in ambient air $c_{im}$, mg m$^{-3}$, is an averaged mass of particles with aerodynamic diameters of 100 μm or less collected from the air drawn through a filter in a high-volume sampler over the sampling period of the test in proximity to the *i*-th stone quarry. The testing was organized at a distance of (1-3) km from each quarry during the quarry' work. Each test lasted 24 hours for collection of particles from about 2000 m$^3$ of air (EPA IO-2.1, 1999). The distribution of the test/measurement results $c_{im}$ at the actual concentration $c_i$ was found to be normal with standard deviation equal to the standard measurement uncertainty $u_i$ =

0.07 $c_{\mathrm{im}}$ and mean equal to $c_i$ (Kuselman et al., 2012a). Corresponding likelihood functions are normal probability density functions (pdfs):

$$f(c_{\mathrm{im}}|c_i) = \frac{1}{u_i\sqrt{2\pi}} \exp\left[-\frac{(c_{im} - c_i)^2}{2u_i^2}\right]. \tag{3}$$

### *2.1.2. Database and prior distributions of actual* concentration *values*

The database of 496 test results obtained during a year and described in the work of Kuselman et al. (2012a) is considered again in the present paper. On the basis of the analysis of variances (ANOVA), it was shown that the wind from the desert did not influence the test results significantly, whereas anthropogenic contributions to TSPM concentration were dominant. No correlation among test results for different quarries was observed. The theoretical distributions of actual values of TSPM concentration $c_i$, fitting successfully the data collected close to quarry $i$, were lognormal distributions, used in the following as prior pdfs:

$$f(c_i) = \frac{1}{c_i\sigma_i\sqrt{2\pi}} \exp\left[-\frac{(\ln c_i - \mu_i)^2}{2\sigma_i^2}\right], \tag{4}$$

where standard deviations $\sigma_i$ and means $\mu_i$ are for the first quarry ($i$ = 1) 0.434 and -2.326, respectively, on the logarithmic scale; for the second quarry ($i$ = 2) they are 0.280 and -2.031, respectively; and for the third quarry $\sigma_3$ = 0.403 and $\mu_3$ = -2.338.

## 2.2. Regulation and acceptance limits

There are national regulations of ambient air quality including upper regulation limits $T_{\mathrm{U}i}$ for TSPM concentration depending on the period of sampling. In Israel, $T_{\mathrm{U}i}$ = 0.200 mg m$^{-3}$ for 24 hours, i.e. the same limit value is valid for any location in the country, also close to the *i*-th quarry.

Besides the regulation limit, a lower/stricter acceptance limits $A_i$ could be applied for the test results with the purpose of decreasing the underestimation (inhabitant's) risks due to measurement uncertainty $u_i$. In such a case, the decision rules (is the air conforming or not?) are based on comparing the test results with the relevant *i*-th acceptance limit (JCGM 106, 2012; Ellison and Williams, 2007). The acceptance limits in the present study are taken as coincidental with the regulation limits.

## 2.3. Particular risks of under- and overestimation

### *2.3.1. Particular specific risks*

The particular specific risks of the pollutant concentration under- and overestimation are respectively

$$R^*_{ci(u)} = \int_{T_{\mathrm{U}i}}^{\infty} f(c_i|c_{im})\,\mathrm{d}c_i,\ \text{for } c_{\mathrm{i}m} \le T_{\mathrm{U}i}, \text{ and} \tag{5a}$$

$$R^*_{ci(\mathrm{o})} = \int_{0}^{T_{\mathrm{U}i}} f(c_i|c_{im})\,\mathrm{d}c_i,\ \text{for } c_{\mathrm{i}m} > T_{\mathrm{U}i}, \tag{5b}$$

where $f(c_i|c_{im})$ is the posterior pdf for the actual value of the TSPM concentration $c_i$ contributed by the $i$-th quarry, given the measurement result near the quarry $c_{i\mathrm{m}}$. From Bayes Law the posterior pdf is

$$f(c_i|c_{im}) = f(c_{i\mathrm{m}}|c_i)f(c_i)/\int_{-\infty}^{\infty} f(c_{im}|c_i)f(c_i)\,\mathrm{d}c_i, \tag{5c}$$

where $f(c_{i\mathrm{m}}|c_i)$ is the likelihood function by eqn (3) and $f(c_i)$ is the prior pdf by eqn (4).

*2.3.2. Particular global risks*

The global risks of $c_i$ under- and overestimation related to the TSPM regulation limit $T_{\mathrm{U}i}$, are respectively

$$R_{ci(\mathrm{u})} = \int_{T_{\mathrm{U}i}}^{\infty} \int_{0}^{T_{\mathrm{U}i}} f(c_{i\mathrm{m}}|c_i)f(c_i)\,\mathrm{d}c_{im}\mathrm{d}c_i, \tag{6a}$$

$$R_{ci(\mathrm{o})} = \int_{0}^{T_{\mathrm{U}i}} \int_{T_{\mathrm{U}i}}^{\infty} f(c_{i\mathrm{m}}|c_i)f(c_i)\,\mathrm{d}c_{im}\mathrm{d}c_i. \tag{6b}$$

*2.3.3. Probabilities of an acceptable test result and a conforming actual concentration value*

Probability $P(\mathrm{C}_i)$ of a conforming test/measurement result for the $i$-th pollutant ($c_{i\mathrm{m}} \leq A_i = T_{\mathrm{U}i}$) is calculated by marginalization of the joint pdf of the measurement results and the actual values of TSPM concentration:

$$P(\mathrm{C}_i) \ = \int_0^\infty \int_0^{T_{\mathrm{U}i}} f(c_{i\mathrm{m}}|c_i) f(c_i)\, \mathrm{d}c_{im} \mathrm{d}c_i \ . \qquad (7a)$$

Probability $P(\overline{\mathrm{B}}_i)$ that the actual concentration value for the $i$-th pollutant is conforming ($c_i \leq T_{\mathrm{U}i}$) is calculated as:

$$P(\overline{\mathrm{B}}_i) \ = \int_0^{T_{\mathrm{U}i}} f(c_i)\, \mathrm{d}c_i \ . \qquad (7b)$$

Note that the probability $P(\overline{\mathrm{B}}_i)$ of a conforming actual (true) value $c_i$ in eqn (7b) does not depend on the measurement result $c_{im}$ by definition. However, the vice versa holds: probability $P(\mathrm{C}_i)$ of a conforming measurement result $c_{im}$ by eqn (7a) does depend on the relevant actual value $c_i$.

# 3. Modeling and calculation

## 3.1. Total risks of overestimation

### *3.1.1. Events*

Define the following events possible during testing concentrations of two or more pollutants in an environmental compartment:

- $\overline{\mathrm{B}}_i$: the actual concentration $c_i$ of pollutant $i$ does not exceed its regulation limit $T_{\mathrm{U}i}$; probability of this event $P(\overline{\mathrm{B}}_i)$ is defined by formula (7b).

- $\overline{\mathrm{B}}$: the actual concentration values $c_i$ for any $i$ do not exceed their own regulation limits $T_{\mathrm{U}i}$, $\overline{\mathrm{B}} = \overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \ldots \cap \overline{\mathrm{B}}_n$; probability of this event is $P(\overline{\mathrm{B}}) = \prod_{i=1}^{n} P(\overline{\mathrm{B}}_i)$, if $\overline{\mathrm{B}}_i$ are mutually independent.
- $\mathrm{B}_i$: the actual concentration $c_i$ of pollutant $i$ exceeds $T_{\mathrm{U}i}$, i.e. violates it; probability of this event is $P(\mathrm{B}_i) = 1 - P(\overline{\mathrm{B}}_i)$.
- B: the actual concentration values $c_i$ of one or more pollutants exceed their regulation limits $T_{\mathrm{U}i}$, $\mathrm{B} = \mathrm{B}_1 \cup \mathrm{B}_2 \cup \ldots \cup \mathrm{B}_n$; probability of this event is $P(\mathrm{B}) = 1 - P(\overline{\mathrm{B}}) = 1 - \prod_{i=1}^{n} P(\overline{\mathrm{B}}_i)$.
- $\mathrm{C}_i$: the test result $c_{i\mathrm{m}}$ for $i$-th pollutant does not exceed its acceptance limit $A_i$; probability of this event $P(\mathrm{C}_i)$ is defined by formula (7a).
- C: the test results $c_{i\mathrm{m}}$ for any $i$ do not exceed their own acceptance limits $A_i$, $\mathrm{C} = \mathrm{C}_1 \cap \mathrm{C}_2 \cap \ldots \cap \mathrm{C}_n$; probability of this event is $P(\mathrm{C}) = \prod_{i=1}^{n} P(\mathrm{C}_i)$, if $\mathrm{C}_i$ are mutually independent.
- $\overline{\mathrm{C}}_i$: the test result $c_{i\mathrm{m}}$ for $i$-th pollutant exceeds its acceptance limit $A_i$ , i.e. such $c_{i\mathrm{m}}$ is an out-of-specification test result (Kuselman et al., 2012b) as $A_i = T_{\mathrm{U}i}$ in the present study; probability of this event is $P(\overline{\mathrm{C}}_i) = 1 - P(\mathrm{C}_i)$.
- $\overline{\mathrm{C}}$: one or more test results $c_{i\mathrm{m}}$ exceed their own $A_i$ , $\overline{\mathrm{C}} = \overline{\mathrm{C}}_1 \cup \overline{\mathrm{C}}_2 \cup \ldots \cup \overline{\mathrm{C}}_n$; probability of this event is $P(\overline{\mathrm{C}}) = 1 - P(\mathrm{C}) = 1 - \prod_{i=1}^{n} P(\mathrm{C}_i)$.

### *3.1.2. Total specific risk*

When a specified environmental compartment is tested concerning concentrations of three pollutants, the total specific risk of overestimation $R^{*}_{\mathrm{total(o)}}$ is the probability that the actual concentrations of all pollutants in this compartment conform to their regulation limits ($\overline{\mathrm{B}} = \overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3$), whereas one or more test/measurement results $c_{1\mathrm{m}}$, $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ exceed their acceptance limits. This event can occur when:

a) Just one measurement result out of the three, for example $c_{1\mathrm{m}}$ without losing generality, exceeds its acceptance limit, while the actual concentration $c_1$ does not exceed the regulation limit. In this case, the actual concentration $c_1$ will be overestimated. Hence, the total risk that the compartment is falsely considered as not conforming is equal to the particular specific risk concerning the first pollutant: $R^*_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}_1|c_{1\mathrm{m}})$.

b) Two measurement results, e.g. $c_{1\mathrm{m}}$ and $c_{2\mathrm{m}}$, exceed their acceptance limits. The total risk is $R^*_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2|c_{1\mathrm{m}}, c_{2\mathrm{m}})$.

c) All the three measurement results exceed their acceptance limits. The total risk is $R^*_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}|c_{1\mathrm{m}}, c_{2\mathrm{m}}, c_{3\mathrm{m}}) = P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3|c_{1\mathrm{m}}, c_{2\mathrm{m}}, c_{3\mathrm{m}})$.

If the events $\overline{\mathrm{B}}_i$ are conditionally independent, i.e. independent of the measurement results $c_{i\mathrm{m}}$, the total specific risk in each of the three considered situations is, respectively:

a) $R^*_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}_1|c_{1\mathrm{m}})$, (8a)

b) $R^*_{\mathrm{total(o)}} = \prod_{i=1}^{2} P(\bar{B}_i|c_{i\mathrm{m}})$, (8b)

c) $R^*_{\mathrm{total(o)}} = \prod_{i=1}^{3} P(\bar{B}_i|c_{i\mathrm{m}})$, (8c)

where $P(\bar{B}_i|c_{i\mathrm{m}}) = R^*_{ci\mathrm{(o)}}$ by formula (5b).

For any number *n* of pollutants, $v \leq n$ of which are characterized by the measurement results exceeding their acceptance limits, the total specific risk of overestimation is

$$R^*_{\mathrm{total(o)}} = \prod_{i=1}^{v} R^*_{ci\mathrm{(o)}}. \qquad (9)$$

Note again that $R^*_{ci\mathrm{(o)}}$ in eqn (9) are related to the out-of-specification measurement results of concentrations of the pollutants, sorted as the first $v$ from all $n$ pollutants under control.

From eqn (9) it follows that any one of $v$ particular specific risk of overestimation $R^*_{ci\mathrm{(o)}}$ equal to zero will lead to $R^*_{\mathrm{total(o)}} = 0$. That occurs when the actual concentration of the $i$-th pollutant $c_i$ exceeds/violates the regulation limit unquestionably ($c_i > T_{\mathrm{U}i}$) at a given measurement result $c_{i\mathrm{m}} > T_{\mathrm{U}i}$ for this pollutant. In such a case, which does not depend on measurement results of concentrations of the other pollutants, the compartment as a whole is certainly not conforming. Therefore, the producer(s) should take action to reduce the $i$-th pollutant concentration and/or to pay a fine.

In the opposite case of a particular specific risk value $R^*_{ci\mathrm{(o)}} = 1$, although $c_{i\mathrm{m}}$ exceeds its acceptance limit, the actual concentration $c_i$ certainly conforms. Such $R^*_{ci\mathrm{(o)}}$ would not influence the total specific risk $R^*_{\mathrm{total(o)}}$ by eqn (9). In this case, the number $n$ of pollutants is *de-facto* decreased by one.

Another property of eqn (9) is reduction of $R^*_{\mathrm{total(o)}}$ with increasing number $v$ of pollutants for which the measurement results are out-of-specification. The logic is that the more such measurement results, the smaller is the total probability of the overestimation. Thus, the greater is the probability that the compartment as a whole does not conform.

Note also that the model used in the work of Subaric-Leitis (2010) and adopted later in the EURAMET guide (Pendrill et al., 2015) leads to an expression equivalent to eqn (9) when the variables (concentrations of the pollutants in our task) are independent, hence validating the model proposed in the present work.

### 3.1.3. Total global risk

Particular global risk $R_{ci(\mathrm{o})}$ of overestimation for the *i*-th pollutant (*i* = 1, 2, 3) is the probability of false nonconformance when the corresponding test result exceeds its acceptance limit $A_i$, while the actual value does not exceed the regulation limit $T_{\mathrm{U}i}$:

$$R_{ci(\mathrm{o})} = P(\overline{\mathrm{B}}_i \cap \bar{\mathrm{C}}_i). \qquad (10)$$

The total global risk $R_{\mathrm{total(o)}}$ of overestimation is the risk of having the actual concentrations of the three pollutants within their regulation limits $T_{\mathrm{U}i}$, when at least one of test results are outside its acceptance limits (that is outside the three-dimensional domain $A_1 \times A_2 \times A_3$), i.e. $R_{\mathrm{total(o)}} = P(\overline{\mathrm{B}} \cap \bar{\mathrm{C}})$, where

$$\overline{\mathrm{B}} \cap \bar{\mathrm{C}} = \overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap (\bar{\mathrm{C}}_1 \cup \bar{\mathrm{C}}_2 \cup \bar{\mathrm{C}}_3) = (\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_1) \cup (\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_2) \cup (\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_3). \qquad (11)$$

The total global risk of overestimation is thus:

$$R_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_1) + P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_2) + P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_3) - P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_1 \cap \bar{\mathrm{C}}_2) - P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_1 \cap \bar{\mathrm{C}}_3) - P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_2 \cap \bar{\mathrm{C}}_3) + P(\overline{\mathrm{B}}_1 \cap \overline{\mathrm{B}}_2 \cap \overline{\mathrm{B}}_3 \cap \bar{\mathrm{C}}_1 \cap \bar{\mathrm{C}}_2 \cap \bar{\mathrm{C}}_3). \qquad (12)$$

Whenever $\overline{\mathrm{B}}_1$, $\overline{\mathrm{B}}_2$ and $\overline{\mathrm{B}}_3$, as well as $\overline{\mathrm{C}}_1$, $\overline{\mathrm{C}}_2$, and $\overline{\mathrm{C}}_3$, are mutually independent, events $\overline{\mathrm{B}}_1 \cap \overline{\mathrm{C}}_1$, $\overline{\mathrm{B}}_2 \cap \overline{\mathrm{C}}_2$ and $\overline{\mathrm{B}}_3 \cap \overline{\mathrm{C}}_3$ are also independent and equation (12) can be rewritten using notation (10) in the following way:

$$R_{\mathrm{total(o)}} = P(\overline{\mathrm{B}}_2)P(\overline{\mathrm{B}}_3)R_{c1(\mathrm{o})} + P(\overline{\mathrm{B}}_1)P(\overline{\mathrm{B}}_3)R_{c2(\mathrm{o})} + P(\overline{\mathrm{B}}_1)P(\overline{\mathrm{B}}_2)R_{c3(\mathrm{o})} - P(\overline{\mathrm{B}}_3)R_{c1(\mathrm{o})}R_{c2(\mathrm{o})} - P(\overline{\mathrm{B}}_2)R_{c1(\mathrm{o})}R_{c3(\mathrm{o})} - P(\overline{\mathrm{B}}_1)R_{c2(\mathrm{o})}R_{c3(\mathrm{o})} + R_{c1(\mathrm{o})}R_{c2(\mathrm{o})}R_{c3(\mathrm{o})}. \qquad (13)$$

Note that eqn (13) is similar to eqn (2) for the total global risk of underestimation. However, it involves probabilities of different events and different particular risks.

In general, for any number *n* of pollutants

$$R_{\mathrm{total(o)}} = \sum_{i=1}^{n}\Big(\prod_{l\neq i} P(\overline{\mathrm{B}}_l)\Big)R_{ci(\mathrm{o})} - \sum_{i=1}^{n}\sum_{j>i}\Big(\prod_{l\neq i,j} P(\overline{\mathrm{B}}_l)\Big)\Big(\prod_{q=i,j} R_{cq(\mathrm{o})}\Big) + \sum_{i=1}^{n}\sum_{j>i}\sum_{k>j}\Big(\prod_{l\neq i,j,k} P(\overline{\mathrm{B}}_l)\Big)\Big(\prod_{q=i,j,k} R_{cq(\mathrm{o})}\Big) + \cdots + (-1)^{n-2}\sum_{i=1}^{n} P(\overline{\mathrm{B}}_i)\Big(\prod_{q\neq i} R_{cq(\mathrm{o})}\Big) + (-1)^{n-1}\prod_{q=1}^{n} R_{cq(\mathrm{o})}, \qquad (14)$$

where *i*, *j*, *k*, *l* and *q* are subscripts of the pollutant in the range (1, … , *n*).

### 3.2. Calculation

When the likelihood function is a normal distribution and the prior pdf is lognormal, the posterior pdf cannot be easily described by an analytical closed form. Therefore, the posterior pdf was obtained by numerical integration (and subsequent normalization) of the product of the prior and the likelihood. The under- and overestimation particular risks were calculated as the fraction of the (approximated) posterior pdf lying outside/inside the tolerance limit, respectively.

Core code developed in R programming environment for calculation of the risks is reported in Annex A. Calculation of total specific risks of under- and overestimation by eqns (1) and (8), respectively, using corresponding particular specific risk values by eqns (5), is shown in Section A-1. Time spent for calculation of the total specific risks with a regular PC (Intel® Core$^{TM}$ i5-3470 Processor, CPU @ 3.20 GHz, Windows 7 Professional 64 bit) is about one second. While increasing (doubling, for example) the number of the involved components does not affect the calculation time, decreasing the numerical integration parameter (stepsize) from 0.001 to 0.0001, increases the execution time up to 6 seconds.

Calculation of total global risks of under- and overestimation by eqns (2) and (13), respectively, using particular global risk values by eqns (6), probabilities of conforming measurement results by eqn (7a) and probabilities of conforming actual concentration values by eqn (7b), is detailed in Section A-2. Time spent for calculation of the total global risks with the same PC is about 5 seconds. In this case, doubling the number of components doubles the required time, whereas decreasing the integration parameter (step) from 0.00001 to 0.000001 increases the computational time up to about 37 seconds.

## 4. Results and discussion

### 4.1. Total specific risks of under- and overestimation

Dependence of the total specific risks of underestimation of TSPM concentration in air on the measurement results $c_{im}$ is demonstrated in Fig. 2. A case when only the first quarry is active and the total risk $R^*_{\mathrm{total(u)}}$ equals to the particular risk $R^*_{c1(u)}$, is shown in Fig. 2a by solid line 1. Dotted lines 3 and 2 point a measured TSPM concentration $c_{1\mathrm{m}}$ = 0.194 mg m$^{-3}$ and corresponding risk value $R^*_{c1(u)}$ = 0.211, as an instance. One can see in Fig. 2a that $R^*_{c1(u)}$ is close to zero (negligible) at $c_{1\mathrm{m}} < 0.170$ mg m$^{-3}$, however significantly increasing with $c_{1\mathrm{m}}$ approaching the tolerance limit $T_{\mathrm{U1}}$ = 0.200 mg m$^{-3}$.

Fig. 2

A case when only the second and the third quarries are active, is represented in Fig. 2b, where the total risk, $R^*_{\mathrm{total(u)}}$, shown as a surface, depends on both $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ in the range [0.010, 0.200] mg m$^{-3}$. The surface lies mostly on the bottom of the three-dimensional region where $R^*_{\mathrm{total(u)}}$ is close to zero, as in Fig. 2a, increasing with $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ approaching their tolerance limits $T_{\mathrm{U1}} = T_{\mathrm{U2}}$ = 0.200 mg m$^{-3}$. When both $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ simultaneously approach 0.200 mg m$^{-3}$, this leads to a ‘protuberance‘ in the total risk surface.

The same dependence of $R^*_{\mathrm{total(u)}}$ on $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ is observed when all the three quarries are active simultaneously, but $c_{1\mathrm{m}} < 0.170$ mg m$^{-3}$: the contribution of the particular risk $R^*_{c1(u)}$ to the total one in such a case is negligible as shown in Fig. 2a. For comparison, Fig. 2c illustrates a scenario when all the three quarries are active and $R^*_{\mathrm{total(u)}}$ - the surface - is depending on $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ in the range [0.010, 0.200] mg m$^{-3}$ as in Fig. 2b, whereas $c_{1\mathrm{m}}$ = 0.194 mg m$^{-3}$. Fig. 2c seems very similar to Fig. 2b. However, the color scales of the $R^*_{\mathrm{total(u)}}$ surfaces are different, since the scale in Fig. 2c is greater because of the significant contribution of $R^*_{c1(u)}$ = 0.211 at $c_{1\mathrm{m}}$ = 0.194 mg m$^{-3}$ (indicated in Fig 2a by dotted lines).

Dependence of the total specific risks of overestimation of the actual TSPM concentration in air on measurement results, when they are out-of-specification ($c_{i\mathrm{m}} > T_{\mathrm{U}i}$), is detailed in Fig. 3. A case when only the first quarry is active, and the total risk $R^*_{\mathrm{total(o)}}$ is equal to the particular risk $R^*_{c1(\mathrm{o})}$, is shown in Fig. 3a by solid line 1. Dotted lines 3 and 2 point a measured TSPM concentration $c_{1\mathrm{m}}$ = 0.250 mg m$^{-3}$ and corresponding risk value $R^*_{c1(\mathrm{o})}$ = 0.008, as an example. Naturally, the risk of overestimation increases as $c_{1\mathrm{m}}$ approaches 0.200 mg m$^{-3}$ (the tolerance limit), and is close to zero for $c_{1\mathrm{m}} > 0.260$ mg m$^{-3}$.

Fig. 3

The case when only the second and the third quarries are active, as in Fig 2b, and $R^*_{\mathrm{total(o)}}$ value depending on both $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ in the range [0.210, 0.300] mg m$^{-3}$, is shown in Fig. 3b. The maximum $R^*_{\mathrm{total(o)}}$ value is observed as $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ near the tolerance limit simultaneously.

Fig. 3c illustrates a case when all the three quarries are active, as in Fig. 2c, but $c_{1\mathrm{m}}$ = 0.250 mg m$^{-3}$. The scale of the $R^*_{\mathrm{total(u)}}$ surface, shown by the color bar, is two orders less than in Fig. 3b. The reason is that the total risk of overestimation, defined as a product of the three particular risks, is influenced by the contribution of $R^*_{c1(\mathrm{o})}$ = 0.008 at $c_{1\mathrm{m}}$ = 0.250 mg m$^{-3}$ (indicated in Fig 3a by dotted lines). In other words, if an out-of-specification measurement result is significantly greater than the tolerance limit, the probability of violation of the regulation is high and the particular risk of overestimation is low. Therefore the total specific risk of overestimation is low also.

## 4.2. Total global risks of under- and overestimation

The particular global risks of underestimation $R_{c1(\mathrm{u})} = 0.006$, $R_{c2(\mathrm{u})} = 0.010$ and $R_{c3(\mathrm{u})} =$ 0.005 obtained here are equal to the values published earlier (Kuselman et al., 2012a). They are used as a part of the validation process of the current calculations. The probabilities of conforming measurement results are $P(\mathrm{C}_1) = 0.949$, $P(\mathrm{C}_2) = 0.929$ and $P(\mathrm{C}_3) = 0.963$. The total risk of underestimation, evaluated in the present work for the first time, is $R_{\mathrm{total(u)}} = 0.019$, hence greater than the particular risk contributed by each quarry.

The particular global risks of overestimation are $R_{c1(\mathrm{o})} = 0.007$, $R_{c2(\mathrm{o})} = 0.015$ and $R_{c3(\mathrm{o})} = 0.006$. They are also equal to those published by Kuselman et al. (2012a). The probabilities of conforming actual concentration values calculated are $P(\overline{\mathrm{B}}_1) = 0.951$, $P(\overline{\mathrm{B}}_2) = 0.934$ and $P(\overline{\mathrm{B}}_3) = 0.965$. The total risk of overestimation, evaluated in the present work for the first time as well, is $R_{\mathrm{total(o)}} = 0.026$, again greater than each $R_{ci(\mathrm{o})}$.

The total risk of overestimation $R_{\mathrm{total(o)}}$ exceeds the total risk of underestimation $R_{\mathrm{total(u)}}$, which implies that there is a reasonable balance between the requirements of an inhabitant’s quality of life and the producer’s expenditure on environmental protection.

## 5. Conclusions

Quantification of risks of false decisions in conformity assessment of an environmental compartment due to measurement uncertainty of concentrations of two or more pollutants, is developed. Even if the assessment of conformity for each pollutant in the compartment is successful, the total probability of a false decision concerning the compartment as a whole might still be significant.

A model of the total probability of a false decision, formulated on the basis of the law of total probability, is used for a study of test results of total suspended particulate matter concentration in ambient air from three independent stone quarries in Israel. Total probabilities of underestimation of the particulate matter concentration (total risk of the inhabitants) and overestimation (total risk of the stone producers) are evaluated as a combination of the particular risks of air conformity assessment near to each quarry.

It is shown that the total global risk of underestimation of the particulate matter concentration is smaller than the total risk of its overestimation. That is a reasonable balance between the requirements of an inhabitant's quality of life and the producer's expenditure on environmental protection.

## Acknowledgment

This research was supported in part by the International Union of Pure and Applied Chemistry (IUPAC Project 2016-007-1-500).

## Appendix A. Core of the R code

### A-1. Calculation of the total specific risks

```
##############
# Specific risks #
##############

# Input data for the quarries
mu1 = -2.326            # Prior location parameter for Q1
mu2 = -2.031            # Prior location parameter for Q2
mu3 = -2.338            # Prior location parameter for Q3
sigma1 = 0.434          # Prior scale parameter for Q1
sigma2 = 0.280          # Prior scale parameter for Q2
sigma3 = 0.403          # Prior scale parameter for Q3
```

```
Rsigmam = 0.07         # Relative measurement uncertainty
TU = 0.2               # Tolerance limit

# Settings for numerical integrations
stepsize <- 0.001
obsvalues = seq(0.01,TU,stepsize)
postmean = rep(0,length(obsvalues))
poststd  = rep(0,length(obsvalues))
Rspec1  = rep(0,length(obsvalues))
Rspec2  = rep(0,length(obsvalues))
Rspec3  = rep(0,length(obsvalues))
c = seq(0,0.5,stepsize)

####################################################
# Consumer specific risk for each observed value in [0.01, TU]
# Normal Likelihood and Lognormal prior

# Q1
i = 1
prior <- dlnorm(c, meanlog = mu1, sdlog = sigma1)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
 poststd[i] = sqrt(postvar)
 Rspec1[i] = stepsize*sum(posterior[c>TU])
 i = i+1
}

# Q2
i = 1
prior <- dlnorm(c, meanlog = mu2, sdlog = sigma2)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
```

```
 poststd[i] = sqrt(postvar)
 Rspec2[i] = stepsize*sum(posterior[c>TU])
 i = i+1
}

# Q3
i = 1
prior <- dlnorm(c, meanlog = mu3, sdlog = sigma3)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
 poststd[i] = sqrt(postvar)
 Rspec3[i] = stepsize*sum(posterior[c>TU])
 i = i+1
}

# Total specific consumer risk for the particular case obs1=obs2=obs3
Rtotu = Rspec1 + Rspec2 + Rspec3 - Rspec1*Rspec2 - Rspec1*Rspec3 - Rspec2*Rspec3 +
Rspec1*Rspec2*Rspec3

#####################################################
# Producer specific risk for each observed value in [0.21, 0.3]

# Settings for numerical integrations
obsvalues = seq(0.21,0.3,stepsize)
postmean = rep(0,length(obsvalues))
poststd  = rep(0,length(obsvalues))
Rspec1  = rep(0,length(obsvalues))
Rspec2  = rep(0,length(obsvalues))
Rspec3  = rep(0,length(obsvalues))

# Q1
i = 1
prior <- dlnorm(c, meanlog = mu1, sdlog = sigma1)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
```

```
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
 poststd[i] = sqrt(postvar)
 Rspec1[i] = stepsize*sum(posterior[c<=TU])
 i = i+1
}

# Q2
i = 1
prior <- dlnorm(c, meanlog = mu2, sdlog = sigma2)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
 poststd[i] = sqrt(postvar)
 Rspec2[i] = stepsize*sum(posterior[c<=TU])
 i = i+1
}

# Q3
i = 1
prior <- dlnorm(c, meanlog = mu3, sdlog = sigma3)
logprior <- log(prior)
for(obs in obsvalues)
{
 loglik <- dnorm(obs, mean = c, sd = Rsigmam*obs, log = T)
 logpos <- logprior + loglik
 posterior <- exp(logpos)
 posterior <- posterior/(sum(posterior)*stepsize)
 postmean[i] <- sum(posterior*c)*stepsize
 postvar <- sum(posterior*(c^2))*stepsize-postmean[i]^2
 poststd[i] = sqrt(postvar)
 Rspec3[i] = stepsize*sum(posterior[c<=TU])
 i = i+1
}

# Total specific producer risk for the particular case obs1=obs2=obs3
Rtoto = Rspec1*Rspec2*Rspec3
```

### A-2. Calculation of the total global risks

```
#############
# Global risks #
#############

# Input data for the quarries
mu1 = -2.326            # Prior location parameter for Q1
mu2 = -2.031            # Prior location parameter for Q2
mu3 = -2.338            # Prior location parameter for Q3
sigma1 = 0.434          # Prior scale parameter for Q1
sigma2 = 0.280          # Prior scale parameter for Q2
sigma3 = 0.403          # Prior scale parameter for Q3
um = 0.07               # Relative measurement uncertainty
T = 0.2                 # Tolerance limit
A = T                   # Acceptance limit

# Consumer's risk Rc and the producer's risk Rp
# Normal Likelihood and Lognormal prior
# Initializations
step = 0.00001
etac = seq(T,10,step)              # Integral domain [T, infinity]
etap = seq(step,T,step)            # Integral domain [0, T]
etacinf = seq(step,10,step)        # Integral domain [0, infinity]

# Q1
ymeanlogQ1 = mu1
ystdlogQ1 = sigma1
RcQ1 = sum( (pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
dlnorm(etac,ymeanlogQ1,ystdlogQ1) * step)
PC1 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
dlnorm(etacinf,ymeanlogQ1,ystdlogQ1) * step)
RpQ1 = sum( (1-pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ1,ystdlogQ1) * step)
PBcompl1 = plnorm(T,ymeanlogQ1,ystdlogQ1)
c(RcQ1,RpQ1,PC1,PBcompl1)
# [1] 0.005769988 0.007368876 0.949038432 0.950637320

# Q2
ymeanlogQ2 = mu2
ystdlogQ2 = sigma2
RcQ2 = sum( (pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
dlnorm(etac,ymeanlogQ2,ystdlogQ2) * step)
PC2 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
dlnorm(etacinf,ymeanlogQ2,ystdlogQ2) * step)
```

```
RpQ2 = sum( (1- pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ2,ystdlogQ2) * step)
PBcompl2 = plnorm(T,ymeanlogQ2,ystdlogQ2)
c(RcQ2,RpQ2,PC2,PBcompl2)
# [1] 0.01045913 0.01525355 0.92911792 0.93391234

# Q3
ymeanlogQ3 = mu3
ystdlogQ3 = sigma3
RcQ3 = sum( (pnorm((A-etac)/(um*etac)) - pnorm(-etac/(um*etac))) *
dlnorm(etac,ymeanlogQ3,ystdlogQ3) * step)
PC3 = sum( (pnorm((A-etacinf)/(um*etacinf)) - pnorm(-etacinf/(um*etacinf))) *
dlnorm(etacinf,ymeanlogQ3,ystdlogQ3) * step)
RpQ3 = sum( (1- pnorm((A-etap)/(um*etap))) * dlnorm(etap,ymeanlogQ3,ystdlogQ3) * step)
PBcompl3 = plnorm(T,ymeanlogQ3,ystdlogQ3)
c(RcQ3,RpQ3,PC3,PBcompl3)
# [1] 0.004602961 0.006233814 0.963053939 0.964684793

# TOTAL global consumer risk (underestimation risk)
c(PC1,PC2,PC3)
# [1] 0.9490384 0.9291179 0.9630539
c(RcQ1,RcQ2,RcQ3)
# [1] 0.005769988 0.010459133 0.004602961
Rtotu = PC2*PC3*RcQ1 + PC1*PC3*RcQ2 + PC1*PC2*RcQ3 - PC3*RcQ1*RcQ2 -
PC2*RcQ1*RcQ3 - PC1*RcQ2*RcQ3 + RcQ1*RcQ2*RcQ3
Rtotu #  0.01865286, for step = 0.00001

# TOTAL global producer risk (overestimation risk)
c(PBcompl1,PBcompl2,PBcompl3)
# [1] 0.9506373 0.9339123 0.9646848
c(RpQ1,RpQ2,RpQ3)
# [1] 0.007368876 0.015253553 0.006233814
Rtoto =  PBcompl2*PBcompl3*RpQ1 + PBcompl1*PBcompl3*RpQ2 +
PBcompl1*PBcompl2*RpQ3 - PBcompl3*RpQ1*RpQ2 - PBcompl2*RpQ1*RpQ3 -
PBcompl1*RpQ2*RpQ3 + RpQ1*RpQ2*RcQ3
Rtoto # 0.0259206, for step = 0.00001
```

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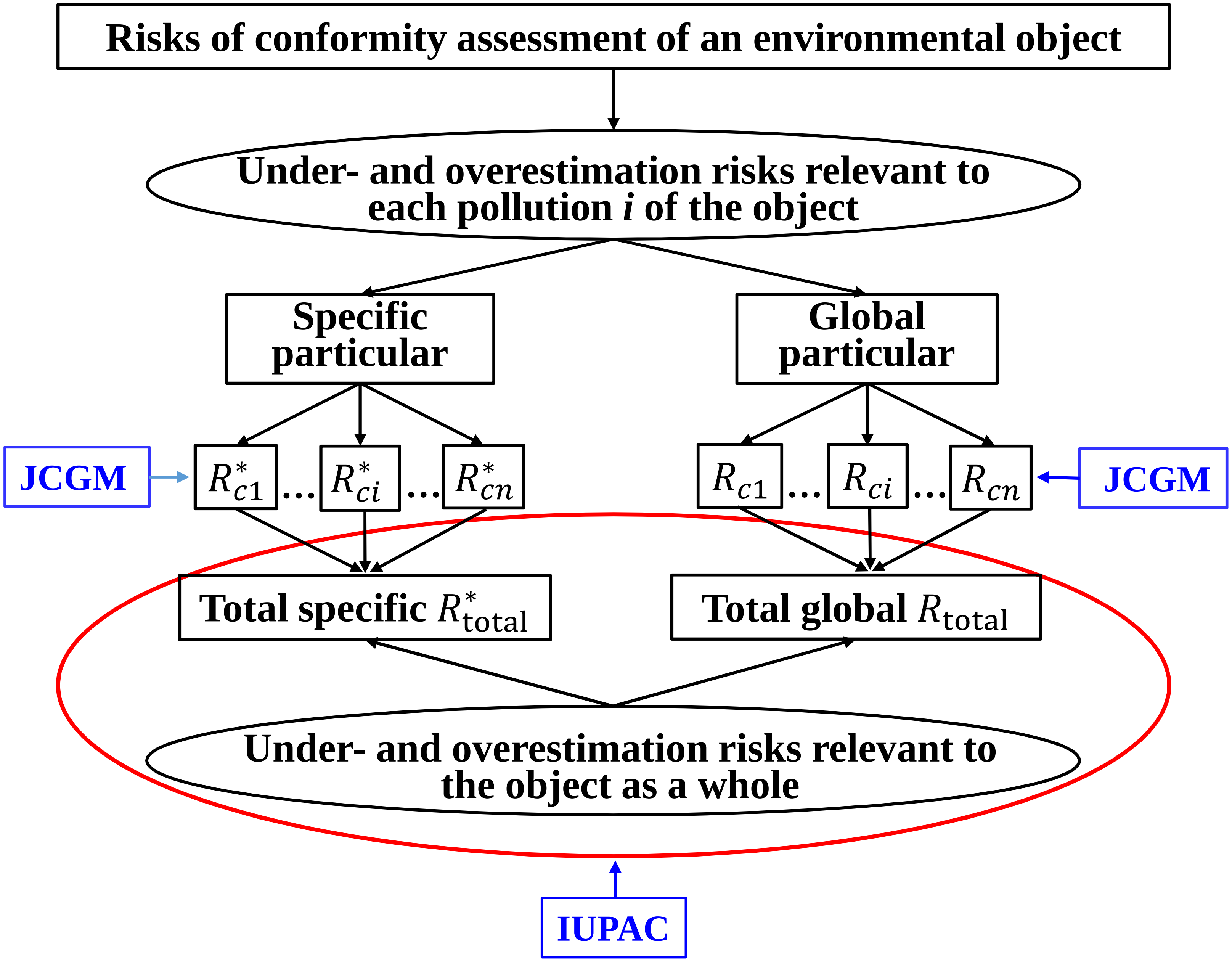
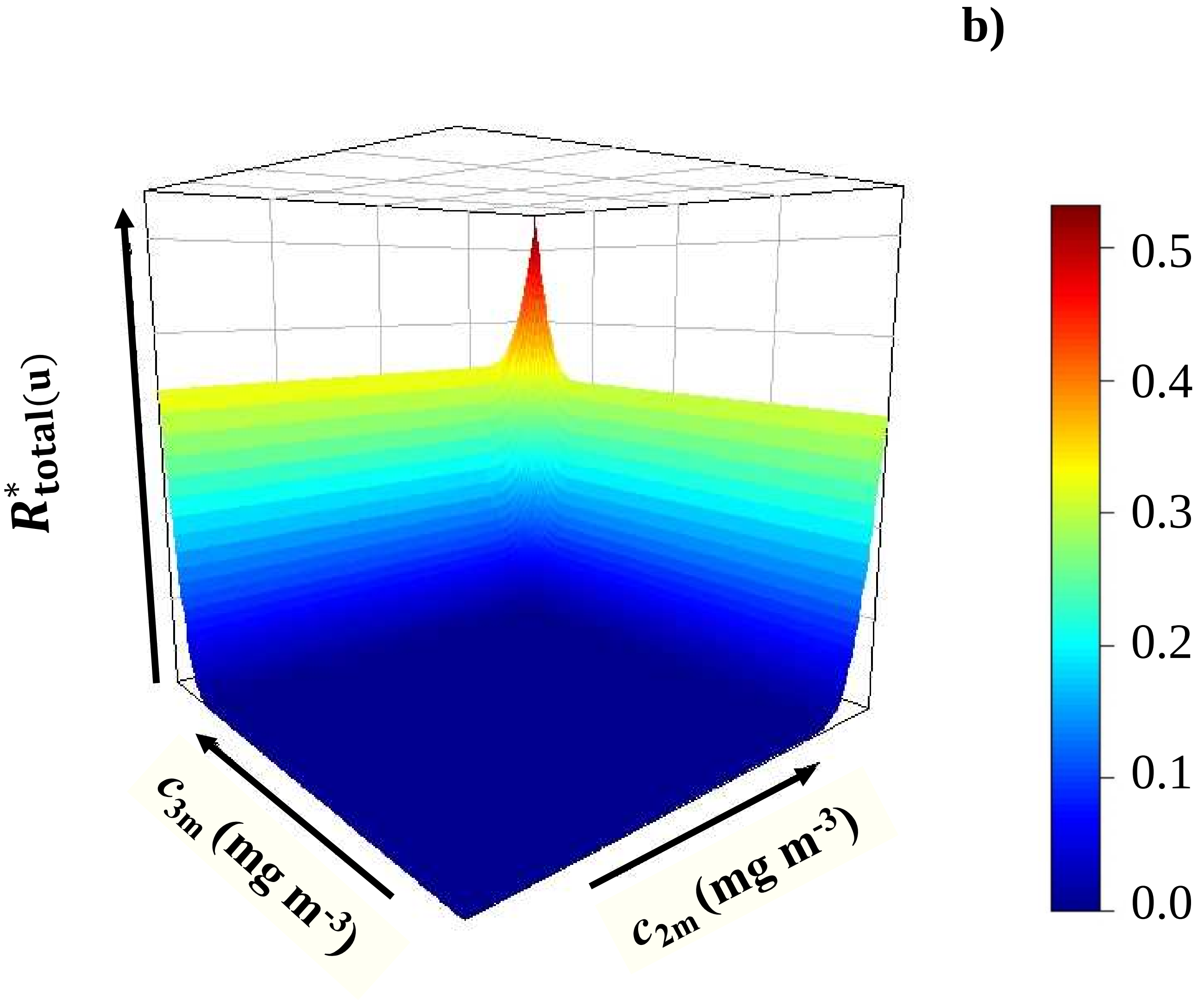
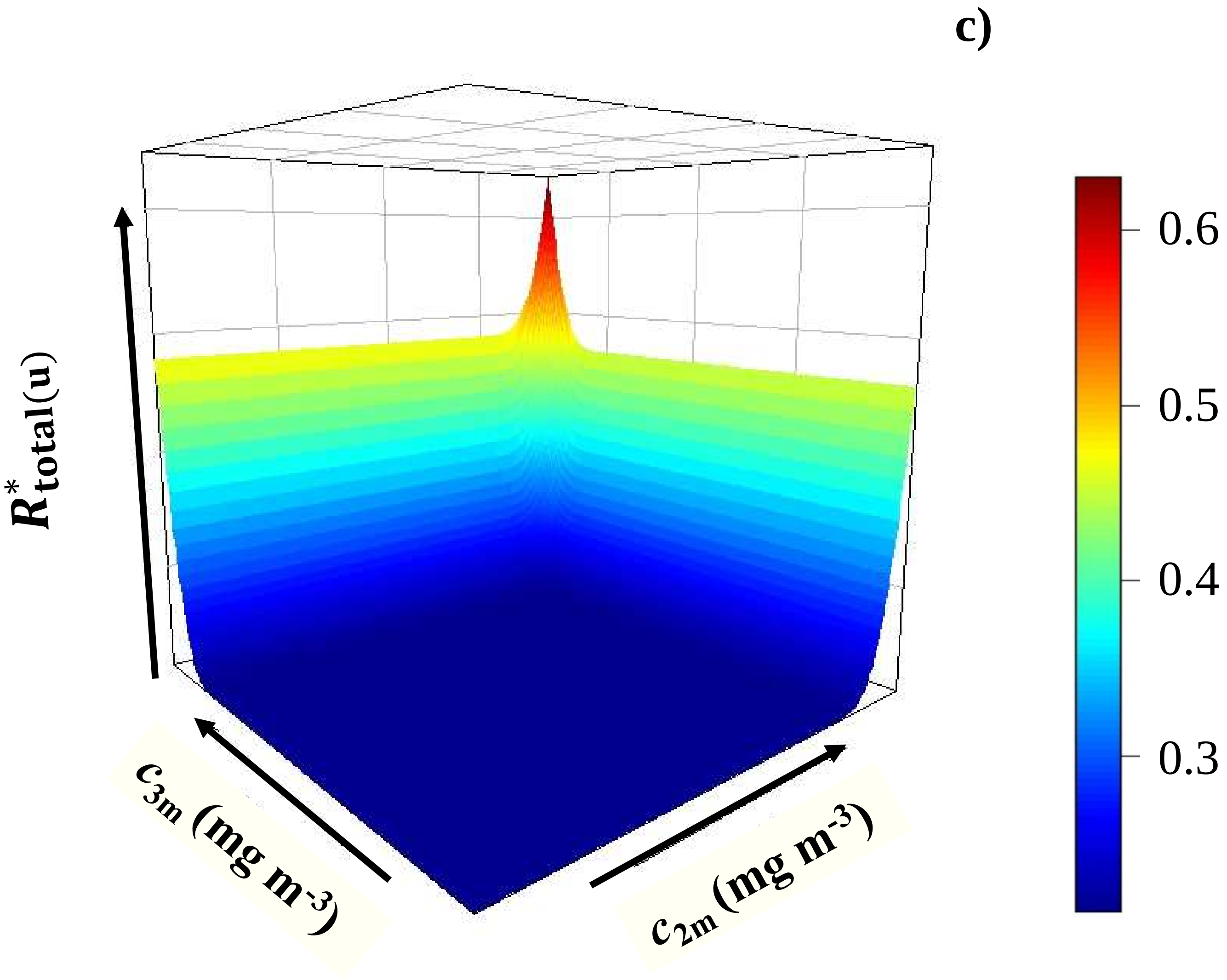
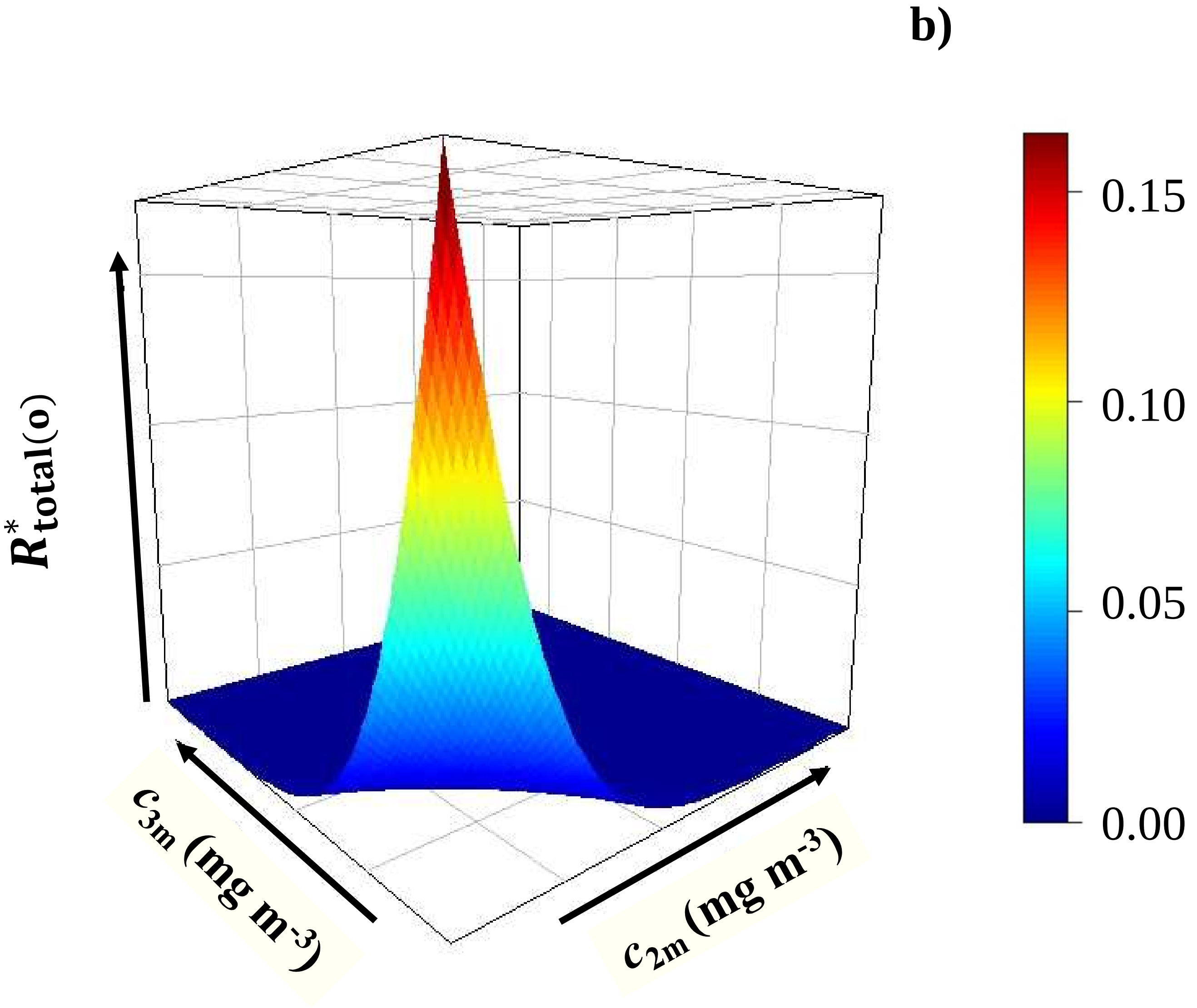
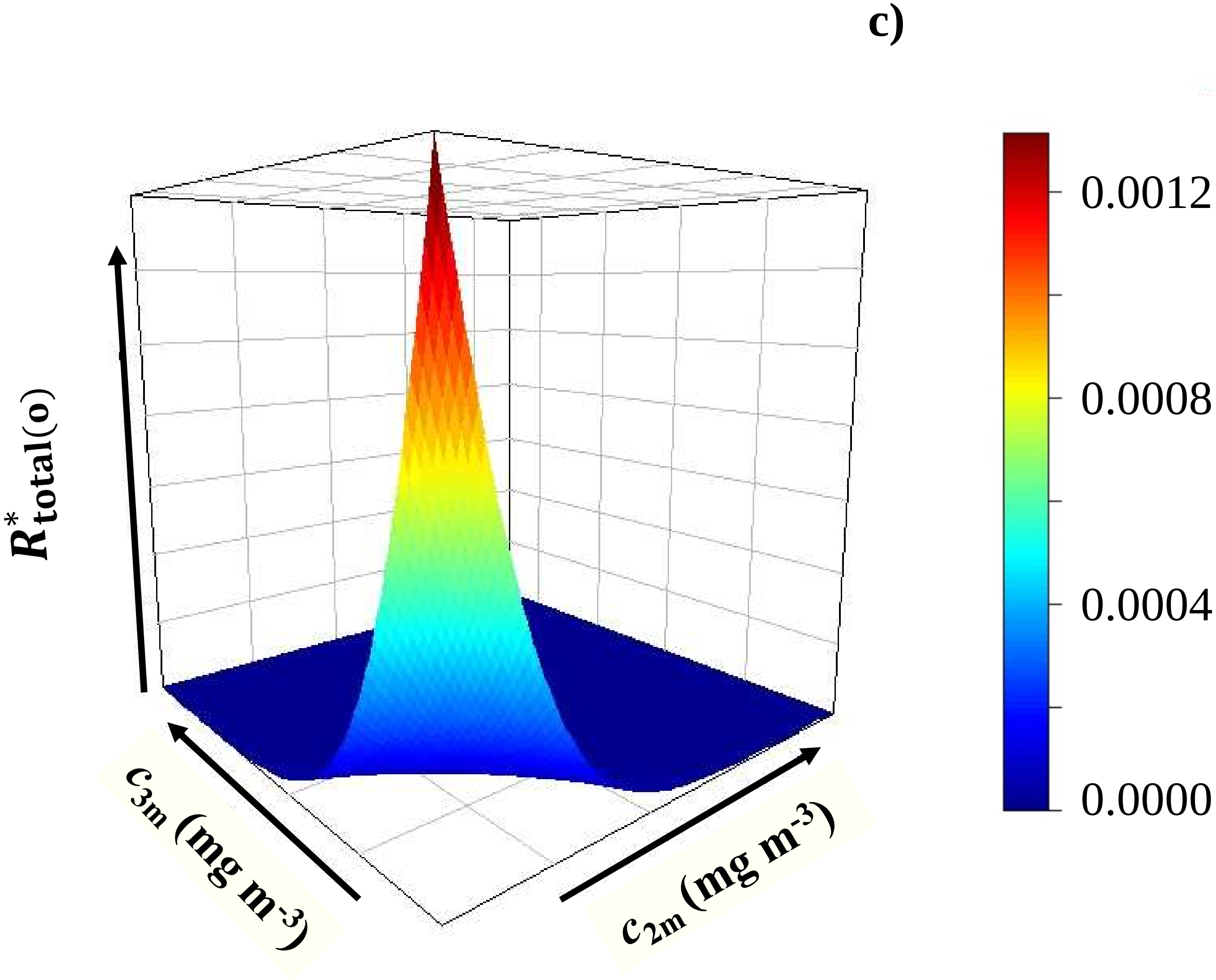
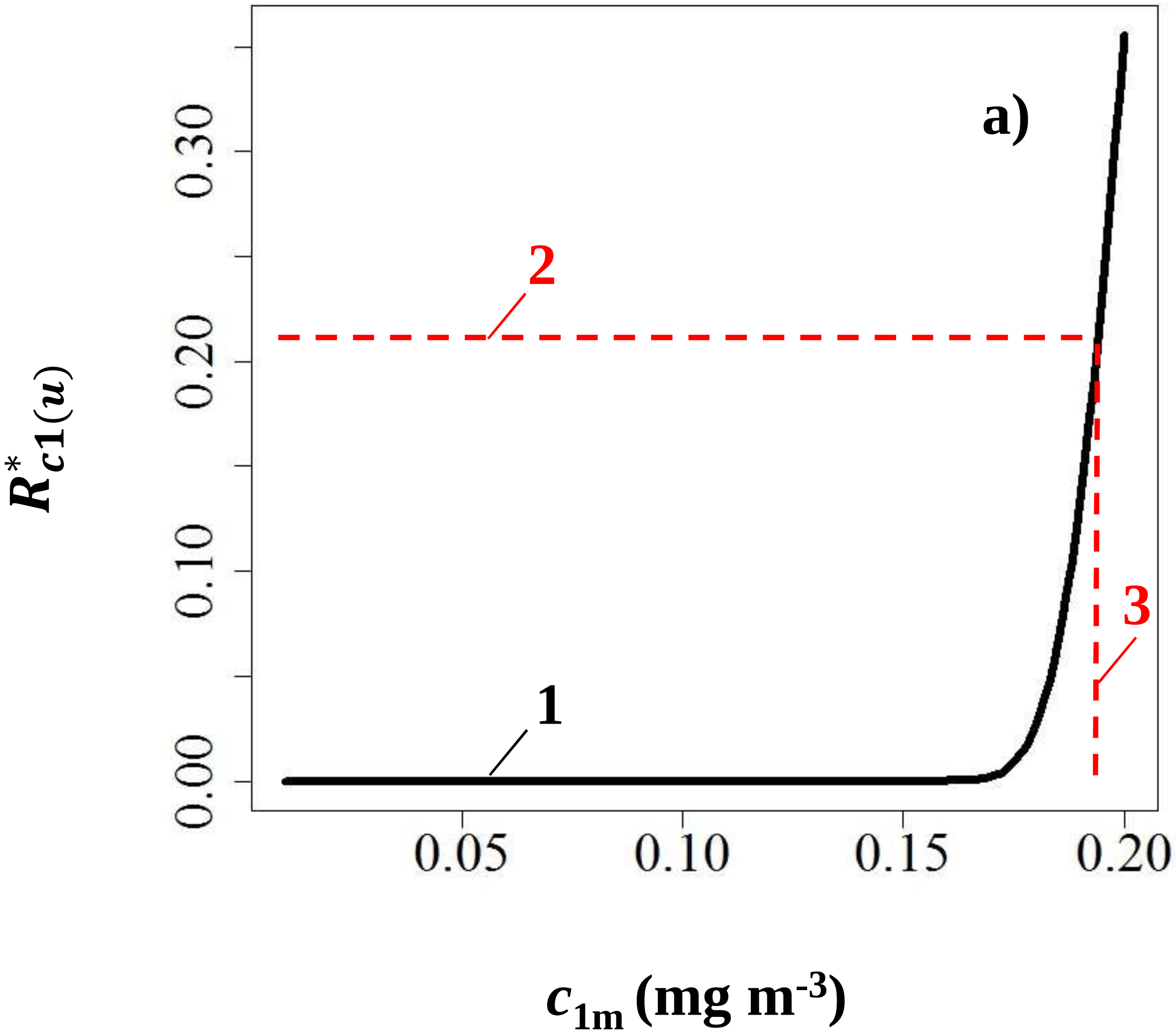

Risks of conformity assessment of an environmental object
Under- and overestimation risks relevant to each pollution *i* of the object
Specific particular
Global particular
JCGM
$R^*_{c1}$ … $R^*_{ci}$ … $R^*_{cn}$
$R_{c1}$ … $R_{ci}$ … $R_{cn}$
JCGM
Total specific $R^*_{\text{total}}$
Total global $R_{\text{total}}$
Under- and overestimation risks relevant to the object as a whole
IUPAC

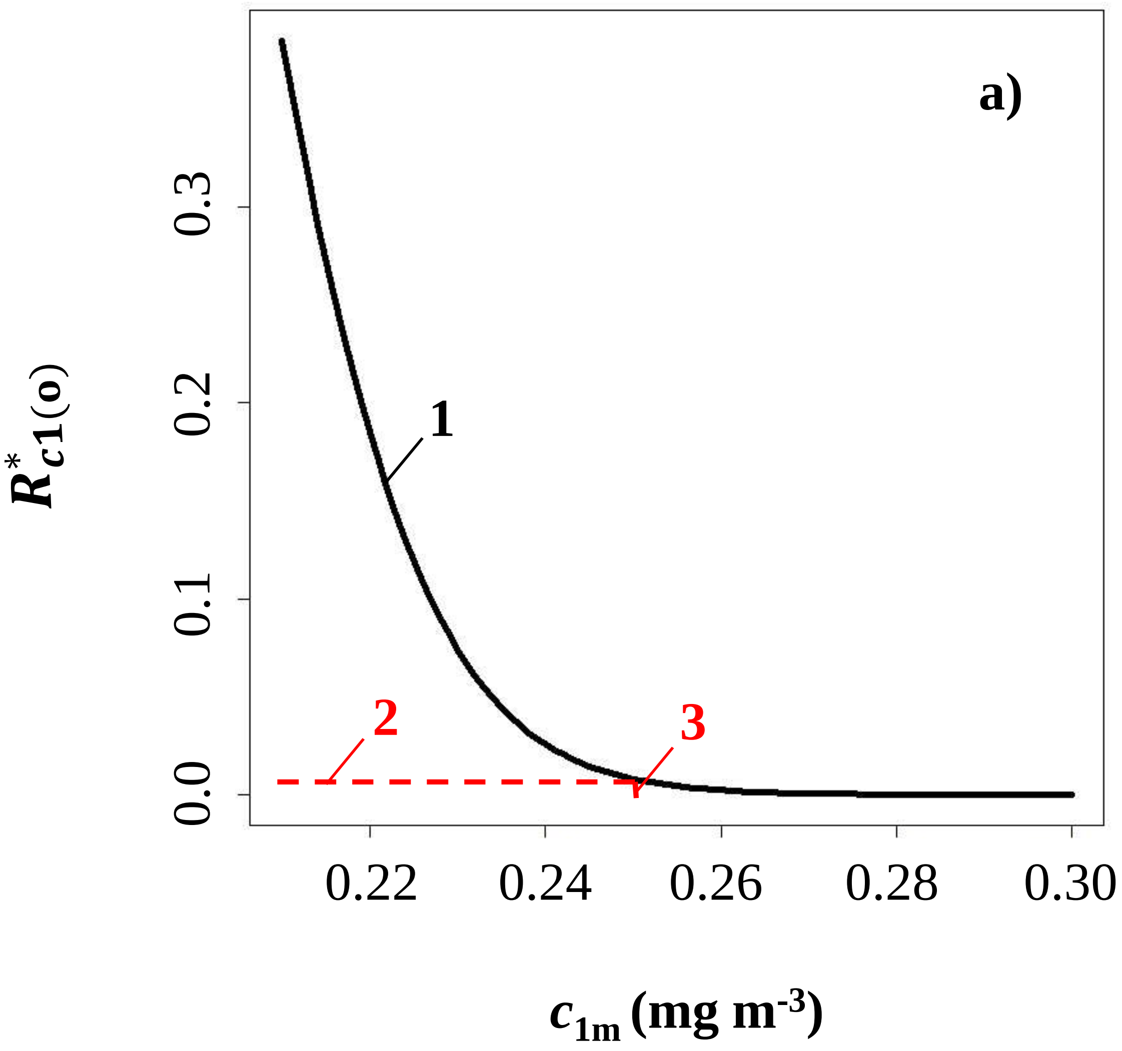

b)
$R^*_{total(u)}$
$c_{3m}$ (mg m$^{-3}$)
$c_{2m}$ (mg m$^{-3}$)
0.5
0.4
0.3
0.2
0.1
0.0


c)
$R^*_{total(u)}$
$c_{3m}$ (mg m$^{-3}$)
$c_{2m}$ (mg m$^{-3}$)
0.6
0.5
0.4
0.3


b)
$R^*_{\mathrm{total(o)}}$
$c_{3m}$ (mg m$^{-3}$)
$c_{2m}$ (mg m$^{-3}$)
0.15
0.10
0.05
0.00


c)
$R^*_{\mathrm{total(o)}}$
$c_{3m}$ (mg m$^{-3}$)
$c_{2m}$ (mg m$^{-3}$)
0.0012
0.0008
0.0004
0.0000


a)
2
3
1
0.00
0.10
0.20
0.30
0.05
0.10
0.15
0.20
$R^*_{c1(u)}$
$c_{1m}$ (mg m$^{-3}$)


a)
1
2
3
0.0
0.1
0.2
0.3
0.22
0.24
0.26
0.28
0.30
$R^*_{c1(o)}$
$c_{1m}$ (mg m$^{-3}$)

## Figure captions

**Fig. 1. Classification of the risks in conformity assessment of an environmental compartment due to measurement uncertainty.** Specific risk refers to a specified compartment in a certain location at a certain time, whereas global risk – to the population of such compartments. Particular risk (specific $R^*_{ci}$ or global $R_{ci}$) refers to *i*-th pollutant of the environmental compartment, $i = 1, 2, \ldots, n$, according to the JCGM Guide 106 (2012); and total risk (specific $R^*_{\text{total}}$ or global $R_{\text{total}}$) – to the compartment as a whole. The total risk evaluation is the task of the IUPAC Project (2016), highlighted in the figure by an ellipse. These kinds of risks are relevant as for an underestimation of the pollutant concentration $c_i$, as for its overestimation, i.e. to the consumer' and producer's risks, respectively.

**Fig. 2. Dependence of the total specific risks of underestimation $R^*_{\text{total(u)}}$ of TSPM concentration in air on the measurement results $c_{i\text{m}}$.** Fig. 2a is for a case when only the first quarry is active and the total risk $R^*_{\text{total(u)}}$ is equal to the particular risk $R^*_{c1(u)}$, shown by solid line 1. Dotted lines 3 and 2 point, as an example, a measured TSPM concentration $c_{1\text{m}} = 0.194$ mg m$^{-3}$ and corresponding risk value $R^*_{c1(u)} = 0.211$. Fig. 2b is for a case when only the second and the third quarries are active. $R^*_{\text{total(u)}}$, presented as a color surface, is depending on both $c_{2\text{m}}$ and $c_{3\text{m}}$ in the range [0.010, 0.200] mg m$^{-3}$. The meaning of the color is the total risk value according to the color scale of the bar on the right side of the plot. Fig. 2c illustrates a case when all the three quarries are active and $R^*_{\text{total(u)}}$ - the color surface - is depending on $c_{2\text{m}}$ and $c_{3\text{m}}$ in the range [0.010, 0.200] mg m$^{-3}$ as in Fig. 2b, but $c_{1\text{m}} = 0.194$ mg m$^{-3}$ (indicated in Fig 2a by dotted lines).

**Fig. 3. Dependence of the total specific risks of overestimation $R^*_{\mathrm{total(o)}}$ of the TSPM concentration in air on the measurement results $c_{i\mathrm{m}}$.** Fig. 3a is for a case when only the first quarry is active and the total risk $R^*_{\mathrm{total(o)}}$ is equal to the particular risk $R^*_{c1(\mathrm{o})}$, shown by solid line 1, while dotted lines 3 and 2 point, as an example, a measured TSPM concentration $c_{1\mathrm{m}}$ = 0.250 mg m$^{-3}$ and corresponding risk value $R^*_{c1(\mathrm{o})}$ = 0.008. Fig. 3b is for a case when only the second and the third quarries are active, as in Fig 2b, and the total risk $R^*_{\mathrm{total(o)}}$ value is depending on both $c_{2\mathrm{m}}$ and $c_{3\mathrm{m}}$ in the range [0.210, 0.300] mg m$^{-3}$. Fig. 3c illustrates a case when all the three quarries are active simultaneously as in Fig. 2c, but $c_{1\mathrm{m}}$ = 0.250 mg m$^{-3}$ (indicated in Fig 3a by dotted lines).

# HIGHLIGHTS

- Evaluation of total risks of false decisions on conformity of an environmental compartment is developed.
- The total risks due to measurement uncertainty of concentrations of two or more pollutants are considered.
- As a case study, the total risks are evaluated at control of total suspended particulate matter (TSPM) concentration in air.
- The study concerns three independent stone quarries as pollutant sources.
- The total probabilities of under- and overestimation of TSPM concentration in air are calculated.