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Conformity assessment of multicomponent materials or objects: Risk of false decisions due to measurement uncertainty - A case study of denatured alcohols

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Abstract: Risk of a false decision on conformity of a multicomponent material or object due to measurement uncertainty is discussed. Even if conformity assessment for each component of a material sample is successful, the total probability of a false decision (total consumer's risk or producer's risk) concerning the sample as a whole might still be significant. A model of the total probability of such false decisions is formulated based on the law (theorem) of total probability. It is shown that the total risk can be evaluated as a combination of the particular risks of conformity assessment of sample components. For a more complicated task, i.e. for a larger number of components of a sample under control, the total risk is greater. As a case study, the total probability of false conforming (total consumer's risk) is evaluated for customs control of completely denatured alcohols, where conformity assessment is performed by comparison of chemical analytical test results with the regulatory limits.

Opposed Reviewers:

Cover Letter

Cover letter

Professor Jean-Michel Kauffmann,

Talanta, Editor

11 September, 2016

Dear Prof. Kauffmann,

Please find attached the manuscript by Ilya Kuselman, Francesca Pennecchi, Ricardo J.N.B. da Silva and D. Brynn Hibbert, titled "Conformity assessment of multicomponent materials or objects: Risk of false decisions due to measurement uncertainty – a case study of denatured alcohols", which we would like to publish in Talanta.

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Best regards,

Dr. Ilya Kuselman,

the manuscript corresponding author,

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*Novelty Statement

Novelty Statement

A new model of the risk of false decisions in conformity assessment of a material or object due to measurement uncertainties is formulated based on the law of total probability, allowing the evaluation of the total risk as a combination of the particular risks of conformity assessment of the material components. The customs risks in assessing denatured alcohols are evaluated as a case study.

*Highlights (for review)

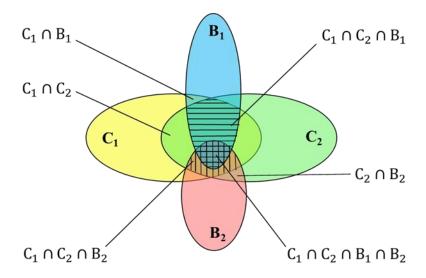
Highlights

- Risk of a false decision on compliance of a multicomponent material is studied
- This risk increases with the number of the material components under control
- A new model of the risk is developed based on the law of total probability
- As a case study the risk of customs control of denatured alcohols is evaluated

*Graphical Abstract (for review)

Graphical abstract

Venn diagram: events C_1 and C_2 , when test results for components 1 and 2, respectively, are in their acceptance intervals, and events B_1 and B_2 , when the true contents of components 1 and 2 are not actually within their tolerance intervals, are shown by ellipses. Other events of interest are indicated as intersections of these ellipses.



Conformity assessment of multicomponent materials or objects: Risk of false decisions due to measurement uncertainty – a case study of denatured alcohols

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ABSTRACT

Risk of a false decision on conformity of a multicomponent material or object due to measurement uncertainty is discussed. Even if conformity assessment for each component of a material sample is successful, the total probability of a false decision (total consumer's risk or producer's risk) concerning the sample as a whole might still be significant. A model of the total probability of such false decisions is formulated based on the law (theorem) of total probability. It is shown that the total risk can be evaluated as a combination of the particular risks of conformity assessment of sample components. For a more complicated task, i.e. for a larger number of components of a sample under control, the total risk is greater. As a case study, the total probability of false conforming (total consumer's risk) is evaluated for customs control of completely denatured alcohols, where conformity assessment is performed by comparison of chemical analytical test results with the regulatory limits.

Keywords

Conformity assessment
Multicomponent object
Measurement uncertainty
Risk of a false decision
Customs control

Denatured alcohols

1. Introduction

The JCGM 106 document [1] provides guidance and procedures for assessing conformity of an item (entity, object or system) with specified requirements. The approach of this document is that knowledge about an item property (the measurand) can be treated as a random variable and expressed in terms of a probability density function (pdf). According to Bayes theorem, such a pdf combines prior knowledge of the measurand and new information acquired during the measurement. The posterior pdf provides an estimate of the measurand value and the associated uncertainty, usually taken as the mean and the standard deviation of the distribution, respectively. Comparing this estimate with the limits of the prescribed tolerance interval of the item property values, one should decide whether the tested item conforms or not [2-4]. It is shown that measurement uncertainty influences the decision and causes risks of two types. The probability of accepting the item, when it should have been rejected, is named 'consumer's risk', whereas the probability of falsely rejecting the item is the 'producer's risk'. For a particular tested item, they are the 'specific consumer's risk' and the 'specific producer's risk' [1, Sec. 9.3.2]. The risks of conformity assessment of an item randomly drawn from a statistical population of such items are the 'global consumer's risk' and the 'global producer's risk' [1, Sec. 9.5], since they characterize the item production globally. These risk terms are taken from the field of product manufacture and process control, but they are also applicable in other fields [5].

Besides the tolerance interval, a narrower acceptance interval (i.e. leading to stronger requirements) for the test results can be applied with the purpose of decreasing the consumer's risk by taking into account the measurement uncertainty. In such a case, the decision rules (is the test item conforming or not?) are based on comparing the measured property values with the acceptance limits [1].

Similar procedures are also described in the earlier Eurachem/CITAC guide [6] for chemical analytical testing laboratories, where items of interest are samples for material analysis, customs control, environmental, food or clinical analysis, etc. The tolerance limits for a sample composition are established specifications in the pharmaceutical industry and other industries and fields, regulatory and/or legislative limits, as well as agreed requirements for a non-regulated product under chemical analysis/testing. In current practice, the decision rules are often based on direct comparison of the measured property values with the specification or regulatory limits.

The reason is that these limits have already taken into account the measurement uncertainty, and so the tolerance and the acceptance limits coincide.

Guidance documents [1] and [6] are widely used, for example, for interpretation of test results of spectral analysis of materials [7], conformity assessment of an analytical instrument [8], forensic decisions on blood alcohol content [9], investigation of out-of-specification test results of chemical composition [10], immunochemical screening of blood donors for infectious diseases [11], in legal metrology [12], in numerous calibration and testing laboratories serving industry and trade. These procedures can be applied where the item is characterized by a single scalar quantity (a single measurable property). In other words, the conformity assessment is performed separately for each item property under testing.

The JCGM 106 approach was extended recently for conformity assessment in presence of a systematic measurement error [13], and for qualitative human-based binary nominal and ordinal properties [14, 15]. A multivariate data analysis is described in the EURAMET guide to decision-making and conformity assessment [16] using bivariate examples of the 'post office parcel problem' (limitations of a parcel length and girth) and a healthcare study (skin cream friction and adhesion).

Multivariate conformity assessment is especially important in testing chemical composition of multicomponent materials or objects where measurement uncertainties are not negligible. When conformity assessments for particular components of a sample are successful and *particular* consumer's risks or producer's risks are acceptable, the total probability of a false decision (*total* consumer's risk or producer's risk) on the conformity of the sample as a whole might still be significant. In this regard, a new IUPAC project [17] was started with the purpose of developing guidelines for evaluating the total probability of a false decision in conformity assessment of a multicomponent material or object caused by measurement uncertainties.

In the present position paper of the project, a model for the total probability of a false decision is formulated based on the law (theorem) of total probability [18-20]. As a case study, the total probability of false conforming (total consumer's risk) is evaluated in customs control of denatured alcohols, where conformity assessment is performed by comparison of chemical analytical test results of denaturants content with regulatory limits. Consequences of the risk (financial, safety, quality and/or others) which are important for the risk management [21-23] are not discussed here.

2. Modeling

Without losing generality, the proposed modeling is focused just on the consumer's risk. The counterpart model for the total producer's risk is easily obtainable.

2.1. Total global risk

Define the following events possible during a sample testing, when the test consists in determining content (concentrations) of two sample components:

- C_1 : the test result (measured property value) c_{1m} for component 1 is in its acceptance interval A_1 ; probability of this event is $P(C_1)$.
- C_2 : the test result c_{2m} for component 2 is in its acceptance interval A_2 ; probability of this event is $P(C_2)$.
- C: the sample as a whole is accepted, i.e. the test results c_{1m} and c_{2m} are in their own acceptance intervals simultaneously, hence $C = C_1 \cap C_2$; probability of this event $P(C) = P(C_1) P(C_2)$, if C_1 and C_2 are mutually independent.
- B_1 : the true content c_1 of component 1 is not within its tolerance interval T_1 ; probability of this event is $P(B_1)$.
- B_2 : the true content c_2 of component 2 is not within its tolerance interval T_2 ; probability of this event is $P(B_2)$.
- B: the sample as a whole is not conforming, i.e. the true content of one or both of the components is not within the corresponding tolerance interval, hence $B = B_1 \cup B_2$; probability of this event is $P(B) = P(B_1) + P(B_2) P(B_1 \cap B_2) = P(B_1) + P(B_2) P(B_1) + P(B_2)$. The last equality is valid if B_1 and B_2 are mutually independent.

Events C_1 and C_2 , as well as B_1 and B_2 , are shown schematically in Fig. 1 by ellipses of a Venn diagram. Other events of interest are indicated as intersections of these ellipses.

Particular global consumer's risk R_{ci} for the *i*-th sample component (i = 1, 2) is the probability of false conformance when the corresponding test result falls within its acceptance limits A_i , while the true value is outside the tolerance limits T_i :

 $R_{c1} = P(C_1 \cap B_1),$ (1)

$$R_{c2} = P(C_2 \cap B_2). \tag{2}$$

At the same time, the total global consumer's risk R_{total} is the risk of having the test results of both the components within their acceptance limits (which are the two-dimensional domain $A_1 \times A_2$), when at least one true content value is outside its tolerance limits T_1 and/or T_2 , i.e. $R_{\text{total}} = C \cap B$, where

$$C \cap B = C_1 \cap C_2 \cap (B_1 \cup B_2) = (C_1 \cap C_2 \cap B_1) \cup (C_1 \cap C_2 \cap B_2). \tag{3}$$

In Fig. 1, event $(C_1 \cap C_2 \cap B_1)$ corresponds to the area shaded by horizontal lines, whereas event $(C_1 \cap C_2 \cap B_2)$ corresponds to the area shaded by vertical lines. The total global consumer's risk is thus:

$$R_{\text{total}} = P(C_1 \cap C_2 \cap B_1) + P(C_1 \cap C_2 \cap B_2) - P(C_1 \cap C_2 \cap B_1 \cap B_2). \tag{4}$$

Event $(C_1 \cap C_2 \cap B_1 \cap B_2)$ is marked in Fig. 1 as a net. Whenever C_1 and C_2 , as well as B_1 and B_2 , are mutually independent, events $C_1 \cap B_1$ and $C_2 \cap B_2$ are also independent and equation (4) can be rewritten using notations (1) and (2) in the following way:

$$R_{\text{total}} = P(C_2)P(C_1 \cap B_1) + P(C_1)P(C_2 \cap B_2) - P(C_1 \cap B_1)P(C_2 \cap B_2)$$

= $P(C_2)R_{c1} + P(C_1)R_{c2} - R_{c1}R_{c2}$. (5)

For example, for particular risks $R_{ci} = 0.05$ and probabilities $P(C_i) = 0.90$, formula (5) gives $R_{total} = 2 \times (0.90 \times 0.05) - 0.05^2 = 0.09$.

For three sample components, under the same assumption of independent true values of each component's content and independent corresponding test results, the total global consumer's risk is:

$$R_{\text{total}} = P(C_2)P(C_3)R_{c1} + P(C_1)P(C_3)R_{c2} + P(C_1)P(C_2)R_{c3} - P(C_3)R_{c1}R_{c2} - P(C_2)R_{c1}R_{c3} - P(C_1)R_{c2}R_{c3} + R_{c1}R_{c2}R_{c3}.$$

$$(6)$$

For example, for particular risks $R_{ci} = 0.05$ and probabilities $P(C_i) = 0.90$, i = 1, 2, 3, formula (6) gives $R_{total} = 3 \times (0.90^2 \times 0.05) - 3 \times (0.90 \times 0.05^2) + 0.05^3 = 0.12$.

For four components, the total global risk is:

$$\begin{split} R_{\text{total}} &= P(C_2)P(C_3)P(C_4)R_{c1} + P(C_1)P(C_3)P(C_4)R_{c2} + P(C_1)P(C_2)P(C_4)R_{c3} + \\ &P(C_1)P(C_2)P(C_3)R_{c4} - P(C_3)P(C_4)R_{c1}R_{c2} - P(C_2)P(C_4)R_{c1}R_{c3} - \\ &P(C_2)P(C_3)R_{c1}R_{c4} - P(C_1)P(C_4)R_{c2}R_{c3} - P(C_1)P(C_3)R_{c2}R_{c4} - \\ &P(C_1)P(C_2)R_{c3}R_{c4} + P(C_4)R_{c1}R_{c2}R_{c3} + P(C_3)R_{c1}R_{c2}R_{c4} + P(C_2)R_{c1}R_{c3}R_{c4} + \\ &P(C_1)R_{c2}R_{c3}R_{c4} - R_{c1}R_{c2}R_{c3}R_{c4} \,. \end{split}$$

For particular risks R_{ci} = 0.05 and probabilities $P(C_i)$ = 0.90, i = 1, 2, 3, 4, by formula (7) one obtains R_{total} = $4 \times (0.90^3 \times 0.05) - 6 \times (0.90^2 \times 0.05^2) + 4 \times (0.90 \times 0.05^3) - 0.05^4 = 0.13$. Comparing this result with the total global risk values for the previous cases of two and three components, it is easy to see that the risk is greater for a larger number of the components under control.

In general, the expression for the total global consumer's risk for a number n of components of a sample under control is:

$$R_{\text{total}} = \sum_{i=1}^{n} (\prod_{l \neq i} P(C_{l})) R_{ci} - \sum_{i=1}^{n} \sum_{j > i} (\prod_{l \neq i, j} P(C_{l})) (\prod_{q = i, j} R_{cq}) +$$

$$\sum_{i=1}^{n} \sum_{j > i} \sum_{k > j} (\prod_{l \neq i, j, k} P(C_{l})) (\prod_{q = i, j, k} R_{cq}) + \dots + (-1)^{n-2} \sum_{i=1}^{n} P(C_{i}) (\prod_{q \neq i} R_{cq}) +$$

$$(-1)^{n-1} \prod_{q=1}^{n} R_{cq} ,$$
(8)

where i, j, k, l and q are indices of the sample components in the range (1, ..., n). Thus, the total global consumer's risk can be evaluated as a combination of n particular global risks of conformity assessment of any material or object in which n components are tested.

2.2. Total specific risk

When a specific batch is tested concerning content of two components, total specific risk R_{total}^* is probability $P(B|c_{1m},c_{2m})$ that the true content of one or both the components in a sample taken from this batch are not within the corresponding tolerance interval ($B = B_1 \cup B_2$), whereas the test/measurement results c_{1m} and c_{2m} of both the components are within their acceptance limits.

If the events B_1 and B_2 are conditionally independent [20, p. 57], i.e. independent at the measurement results c_{1m} and c_{2m} , the total specific risks is

$$R_{\text{total}}^* = P(B|c_{1m}, c_{2m}) = P(B_1 \cup B_2|c_{1m}, c_{2m}) = P(B_1|c_{1m}, c_{2m}) + P(B_2|c_{1m}, c_{2m}) - P(B_1 \cap B_2|c_{1m}, c_{2m}) = P(B_1|c_{1m}) + P(B_2|c_{2m}) - P(B_1|c_{1m})P(B_2|c_{2m}).$$
(9)

Since particular specific consumer's risks R_{ci}^* for the *i*-th component, i = 1, 2, are:

$$R_{c1}^* = P(B_1|c_{1m}), (10)$$

$$R_{c2}^* = P(B_2|c_{2m}), (11)$$

substituting formulas (10) and (11) into formula (9) gives the following:

$$R_{\text{total}}^* = R_{c1}^* + R_{c2}^* - R_{c1}^* R_{c2}^*. \tag{12}$$

For example, for particular specific risks $R_{ci}^* = 0.05$, the total risk by formula (12) is $R_{\text{total}}^* = 2 \times 0.05 - 0.05^2 = 0.10$.

Total specific consumer's risk for three components is:

$$R_{\text{total}}^* = R_{c1}^* + R_{c2}^* + R_{c3}^* - R_{c1}^* R_{c2}^* - R_{c1}^* R_{c3}^* - R_{c2}^* R_{c3}^* + R_{c1}^* R_{c2}^* R_{c3}^* . \tag{13}$$

For example, when the particular specific risks are $R_{ci}^* = 0.05$, i = 1, 2, 3, the total risk by formula (13) is $R_{total}^* = 3 \times 0.05 - 3 \times 0.05^2 + 0.05^3 = 0.14$.

When four components are under control, the total specific risk is:

$$R_{\text{total}}^* = R_{c1}^* + R_{c2}^* + R_{c3}^* + R_{c4}^* - R_{c1}^* R_{c2}^* - R_{c1}^* R_{c3}^* - R_{c1}^* R_{c4}^* - R_{c2}^* R_{c3}^* - R_{c2}^* R_{c4}^* - R_{c3}^* R_{c4}^* + R_{c1}^* R_{c2}^* R_{c3}^* + R_{c1}^* R_{c2}^* R_{c4}^* + R_{c1}^* R_{c3}^* R_{c4}^* + R_{c2}^* R_{c3}^* R_{c4}^* - R_{c1}^* R_{c2}^* R_{c3}^* R_{c4}^*$$

$$(14)$$

For example, when the particular risks are again $R_{ci}^* = 0.05$, i = 1, 2, 3, 4, formula (14) gives $R_{total}^* = 4 \times 0.05 - 6 \times 0.05^2 + 4 \times 0.05^3 - 0.05^4 = 0.19$. Thus, as for the total global risk values, the total specific risk value is greater for a larger number of the components under control.

In general, the total specific consumer's risk for a number n of components is:

$$R_{\text{total}}^* = \sum_{i=1}^n R_{ci}^* - \sum_{i=1}^n \sum_{j>i} \left(\prod_{q=i,j} R_{cq}^* \right) + \sum_{i=1}^n \sum_{j>i} \sum_{k>j} \left(\prod_{q=i,j,k} R_{cq}^* \right) + \cdots + (-1)^{n-2} \sum_{i=1}^n \left(\prod_{q\neq i} R_{cq}^* \right) + (-1)^{n-1} \prod_{q=1}^n R_{cq}^* ,$$

$$(15)$$

where i, j, k, l and q are indices of the sample components in the range (1, ..., n).

Note, formulas (5), (6), (7) and (8) for calculation of total global risk can be simplified to similar combinations of the particular global risks as for specific risks in formulas (12), (13), (14) and (15) for 2, 3, 4 and n components, respectively, when each probability $P(C_i)$ of acceptance of the test results for component i = 1, 2, ..., n is equal to 1.

Note also that the model used in the work [24], adopted later in the EURAMET guide [16], defined the total specific risk as one minus the probability that all the involved variables lie within a multivariate 'hyper-rectangle' tolerance region (that is one minus the total conformance probability), seen as the intersection of all the variable particular tolerance regions. When the variables are independent, the total conformance probability is given by the product of the particular conformance probabilities. It can be shown that this model leads to expressions equivalent to formulas (12) - (15) above, hence validating the model proposed in the present work.

2.3. Cases of interdependence of the events

It is not always possible to assert independence of C₁ and C₂, as well as of B₁ and B₂. A number of chemical analytical techniques are used at method development to overcome possible correlations between measurement results: sample digestion, extraction of analytes from a sample, separation of an analyte from other components, etc. Chemometrics software is applied for separation of spectral signals. Standard additions of an analyte to a sample are used for calibration of a measurement system overcoming multiplicative matrix effects of a sample, and so on. There are requirements in validation guidelines, e.g. [25-27], for evaluating the method selectivity and/or specificity. An experimental proof is necessary that the response of the measurement system is caused by the analyte/component proper, not by another component or

the sample matrix. Still something may happen in practice, but in general this kind of correlation should be negligible.

Correlation of true values of content of different sample components may be caused by stoichiometry of native compounds (in geological, environmental and other samples). The law of conservation of mass means also interdependence of the true concentration values of the components in a sample (their sum must be 100 %). Technological reasons in production of some artificial materials (alloys, drugs, etc.) lead to such correlations as well.

Metrologically-independent test results for two or more components are however inevitably correlated when their true values are correlated. In other words, if B_1 and B_2 are interdependent, also C_1 and C_2 are, though correlation between test results is weaker because of random measurement errors.

For calculating the total consumer's risk taking into account possible correlation between true values of content of components in a sample and/or between results of their measurements, one needs to have the joint pdf of events C_i and B_i . In such case the last term of expression (4), for example, can be calculated as

$$P(C_1 \cap C_2 \cap B_1 \cap B_2) = \int_{A_1} \int_{A_2} \int_{T_1} \int_{T_2} g(c_{1m}, c_{2m}, c_1, c_2) dc_1 dc_2 dc_{1m} dc_{2m} , \qquad (16)$$

where g is a multivariate pdf of measured values c_{1m} and c_{2m} and true values c_1 and c_2 of content of components 1 and 2, respectively.

In the Bayesian context [1], the joint pdf can be rewritten as $g(c_{1m}, c_{2m}, c_1, c_2) = h(c_{1m}, c_{2m}|c_1, c_2)$ $g_0(c_1, c_2)$, where h is a bivariate pdf of measured values c_{1m} and c_{2m} , taking into account possible correlation between them, when the true values are c_1 and c_2 (likelihood), whereas g_0 is a bivariate pdf of true values c_1 and c_2 (prior to measurements). Often, as the likelihood, a bivariate normal distribution can be used, having as expectation the mean vector $[c_1, c_2]$ and as covariance matrix the measurement uncertainty matrix:

$$\begin{bmatrix} u^{2}(c_{1m}) & u(c_{1m}, c_{2m}) \\ u(c_{1m}, c_{2m}) & u^{2}(c_{2m}) \end{bmatrix},$$
 (17)

where $u(c_{im})$ is the standard uncertainty of the *i*-th measurement result, and $u(c_{1m}, c_{2m})$ is the covariance between the two.

Principal component analysis (PCA) of the observed multivariate data can be employed to handle effects of covariance on conformance probabilities, as shown in a study of skin cream friction and adhesion described in Deliverable 3.2.4 of the EURAMET guide [16] and references therein.

3. Evaluation of the risks in customs control of denatured alcohols caused by measurement uncertainty

The guidelines of World Customs Organization [28] and European Commission [29, 30] define risk as the potential non-compliance with customs laws. Risk analysis in customs control includes the systematic use of available information to determine how often the defined event may occur, i.e. its likelihood or probability. When substances and/or materials are under customs control, one of such risks is caused by measurement uncertainty of chemical analytical test results.

For example, alcohol (ethanol for human consumption) is subject to excise duties, while denatured alcohols (for industrial use) are not, and the task of the control is to distinguish between them [31]. According to EU Regulation [32], a common procedure for the purpose of completely denaturing alcohol consists of addition of 3 L of isopropyl alcohol (IPA), 3 L of methyl ethyl ketone (MEK) and of 1 g of denatonium benzoate (DB) to 100 L (1 hL) of absolute ethyl alcohol (EtOH). Similar regulations exist in Israel [33], Australia [34] and other countries.

Analytical methods for testing completely denatured alcohol (CDA) include determination of EtOH, IPA and MEK using gas chromatography with flame ionization detection (GC-FID) and determination of DB using high performance liquid chromatography with ultraviolet detection (HPLC-UV). The analytes are separated completely from other components of a sample at the chromatographic conditions of the methods. Relevant internal standards and calibration standards are used for quantification of the analyte concentrations. IPA and MEK concentrations are expressed in L per hL of EtOH (as measured), DB concentration – in g per hL of EtOH. These methods have been recently validated at the Institute for Reference Materials and Measurements (IRMM) with participation of a number of customs laboratories [35]. There is no

evidence of correlation between measured values of the measurands. The standard measurement uncertainties were evaluated in the validation process based on the interlaboratory study: $u_1 = 0.05 \text{ L} \cdot \text{hL}^{-1}$ for IPA, $u_2 = 0.07 \text{ L} \cdot \text{hL}^{-1}$ for MEK, and $u_3 = 0.07 \text{ g} \cdot \text{hL}^{-1}$ for DB. As the measurand is the denaturant concentration in a batch of alcohol, the variation of test results is influenced by inhomogeneity of the batches also [36]. Moreover, the contribution of inhomogeneity is dominant here. Therefore, in practice the following relative standard deviations of test results are set in the report [35] as acceptable: 5 % or $s_{r1} = s_{r2} = 0.05$ in fractions of 1for IPA and MEK, and 10 % or $s_{r3} = 0.10$ for DB.

A decision on conformity assessment can be made using IPA and MEK test results, rather than DB test results [35]. Therefore, an analysis of the customs' risks caused by the measurement uncertainties is discussed below for the two scenarios: when IPA and MEK concentrations only are under control, and when concentrations of all the denaturants (IPA, MEK and DB) are considered. Since the customs authority dealing with CDA is the 'consumer' in this study, the customs' risks caused by the measurement uncertainties are the consumer's risks.

3.1. Evaluation of the total global customs risk

Because denaturing is the process of transformation of absolute ethanol into an undrinkable poisonous mixture of chemicals, the regulatory requirements to the true concentrations c_1 , c_2 and c_3 of the denaturants, IPA, MEK and DB, respectively, are the single lower regulatory limits lrl_1 , lrl_2 and lrl_3 of their tolerance intervals. By regulation [32], IPA and MEK concentrations c_1 and c_2 in a CDA sample shall be not less than $lrl_1 = lrl_2 = 3 \text{ L} \cdot \text{hL}^{-1}$, whereas DB concentration c_3 - not less than $lrl_3 = 1 \text{ g} \cdot \text{hL}^{-1}$.

The *i*-th particular global customs risk, i = 1, 2, 3, can be evaluated by the following formulas [1, 10]:

$$R_{ci} = \int_0^{lrl_i} \int_{lal_i}^{\infty} f(c_{im}|c_i) f(c_i) dc_{im} dc_i, \qquad (18)$$

where lal_i is the lower acceptance limit for the measurement/test results c_{im} , $f(c_i)$ is the pdf of the global distribution of c_i values (the prior pdf), and $f(c_{im}|c_i)$ is the pdf of the distribution of measurement results c_{im} when the true value is c_i (the likelihood function). The lower bound of the outer integral is equal to zero, since concentration of a denaturant is a non-negative property.

The standard measurement uncertainties u_i can be applied as the standard deviations of the distribution of measurement results c_{im} . When a customs laboratory database contains a statistically significant number of test results of CDA batches, the global distribution of true c_i values can be approximated by an empirical distribution of these 'batch-to-batch' results as shown in the Guide [10]. In the current case study the relative standard deviations s_{ri} , taking into account inhomogeneity of the batches, are used for calculating the standard deviations of the global distributions. For simplicity the global distributions are approximated by the following normal distributions:

$$f(c_i) = \frac{1}{s_{ri}\mu_i\sqrt{2\pi}} \exp\left[-\frac{(c_i - \mu_i)^2}{2(s_{ri}\mu_i)^2}\right],$$
 (19)

where μ_i is the mean, and $s_i\mu_i$ is the standard deviation of the global distribution. The measurement distributions (likelihood functions) are taken as normal as well:

$$f(c_{im}|c_i) = \frac{1}{u_i\sqrt{2\pi}} \exp\left[-\frac{(c_{im} - c_i)^2}{2u_i^2}\right].$$
 (20)

In order to apply formula (18), normal distributions truncated at zero should be used both for the prior and the likelihood pdfs, however, for the example under consideration the influence of the truncation is negligible. The results of calculations of the particular global risks R_{ci} in dependence on μ_i values, when a measurement/test result is compared directly with the regulation limit ($lal_i = lrl_i$), are presented in Fig. 2 for IPA and MEK, curves 1 and 2, respectively, and in Fig. 3 - for DB. From Fig. 2, one can notice how greater measurement uncertainties lead to greater risks. For example, in this study at $\mu_1 = \mu_2 = 3.15 \text{ L} \cdot \text{hL}^{-1}$ and $\mu_3 = 1.10 \text{ g} \cdot \text{hL}^{-1}$ the following risks are observed: $R_{c1} = 0.027$ for IPA and $R_{c2} = 0.034$ for MEK at, and $R_{c3} = 0.046$ for DB. They are indicated by dotted lines 3 and 4 for IPA, and 5 and 6 for MEK in Fig. 2; and by dotted lines 2 and 3 for DB in Fig. 3.

The probability $P(C_i)$ of acceptance of a measurement/test results for *i*-th denaturant is calculated by marginalization of the corresponding joint pdf:

$$P(C_i) = \int_0^\infty \int_{lal_i}^\infty f(c_{im}|c_i) f(c_i) dc_{im} dc_i.$$
 (21)

The following results were obtained (again at $lal_i = lrl_i$) when $\mu_1 = \mu_2 = 3.15 \text{ L} \cdot \text{hL}^{-1}$ and $\mu_3 = 1.10 \text{ g} \cdot \text{hL}^{-1}$: $P(C_1) = 0.818$ for IPA, $P(C_2) = 0.808$ for MEK, and $P(C_3) = 0.778$ for DB.

The total global customs risk, in the case of control of IPA and MEK at the above mentioned conditions, is given by formula (5): $R_{\text{total}} = 0.808 \times 0.027 + 0.818 \times 0.034 - 0.027 \times 0.034 = 0.048$. It is greater than each particular risk.

When all the three denaturants (IPA, MEK and DG) are under control at the same conditions, the total global customs risk by formula (6) is $R_{\rm total} = 0.808 \times 0.778 \times 0.027 + 0.818 \times 0.778 \times 0.034 + 0.818 \times 0.808 \times 0.046 - 0.778 \times 0.027 \times 0.034 - 0.808 \times 0.027 \times 0.046 - 0.818 \times 0.034 \times 0.046 + 0.027 \times 0.034 \times 0.046 = 0.066$. This value is greater than that calculated in the case of control on just IPA and MEK.

3.2. Evaluation of the total specific customs risk

When a specific CDA batch is under customs control, the particular specific customs risk value R_{ci}^* for the *i*-th denaturant can be evaluated according to JCGM 106 [1, Sec. A]:

$$R_{ci}^* = \int_0^{lrl_i} f(c_i|c_{im}) \, \mathrm{d}c_i \,, \tag{22}$$

where

$$f(c_i|c_{im}) = \frac{1}{u_{iPost}\sqrt{2\pi}} \exp\left[-\frac{(c_i - \mu_{iPost})^2}{2u_{iPost}^2}\right]$$
(23)

is the posterior pdf for the true values of the i-th denaturant concentration c_i , while the measurement result obtained at the batch testing is c_{im} . When both the prior and likelihood are normal distribution, also the posterior pdf is normal with the following parameters:

$$\mu_{i\text{Post}} = \frac{\mu_i / (s_{\text{r}i}\mu_i)^2 + c_{\text{im}} / u_i^2}{1 / (s_{\text{r}i}\mu_i)^2 + 1 / u_i^2},$$
(24)

$$u_{iPost} = \frac{1}{\sqrt{\frac{1}{(s_{ri}\mu_i)^2 + \frac{1}{u_i^2}}}}.$$
 (25)

The R_{ci}^* values calculated at the same conditions as in the previous section ($\mu_1 = \mu_2 = 3.15 \text{ L} \cdot \text{hL}^{-1}$ and $\mu_3 = 1.10 \text{ g} \cdot \text{hL}^{-1}$), in dependence on measurement results c_{im} within their acceptance interval, are shown in Fig. 4 by lines 1 and 2 for IPA and MEK, respectively; for DB – in Fig. 5 by line 1.

For example, when a customs laboratory reports in a certificate of analysis of a CDA batch the test results c_{1m} = 3.10 L·hL⁻¹ for IPA, also c_{2m} = 3.10 L·hL⁻¹ for MEK, and c_{3m} = 1.05 g·hL⁻¹ for DB, the batch should be recognized as properly denatured according to the regulation [32]. However, there are still the following particular specific customs risks: $R_{c1}^* = 0.014$, $R_{c2}^* = 0.045$, and R_{c3}^* = 0.138. They are shown by dotted lines 3 and 4 for IPA, 5 and 6 for MEK in Fig. 4, and by dotted lines 2 and 3 for DB in Fig. 5. If IPA and MEK only influence the decision on the batch conformity, the total specific risk is $R_{\text{total}}^* = 0.014 + 0.045 - 0.014 \times 0.045 = 0.059$, by formula (12). When all the denaturants are taken into account, the total specific risk is $R_{\text{total}}^* =$ $0.014 + 0.045 + 0.138 - 0.014 \times 0.045 - 0.014 \times 0.138 - 0.045 \times 0.138 + 0.014 \times 0.045 \times 0.138 = 0.014 \times 0.045 \times 0.014 \times 0.01$ 0.188, by formula (13). This value is caused mostly by DB, since R_{c3}^* is significantly larger here than R_{c1}^* and R_{c2}^* . At the same time, consequences of the risk related to the DB concentration are probably less important than other ones, inasmuch as DB is the bitterest chemical compound known and some variations of its concentration do not change the terrible bitter feeling of a person trying to drink CDA. However, this topic is out of the aim of the current study.

4. Conclusion

When separate conformity assessment for each component of a multicomponent material or object is successful, the total probability of a false decision (consumer's risk or producer's risk) concerning the conformity of the material or object as a whole may still be significant. This probability, caused by measurement uncertainties, is larger for a more complicated composition,

i.e. for a greater number of components of the material or object. Mutual correlations of concentrations of the components can influence the risk value.

A model for the total probability of false conforming (consumer's risk) based on the law of total probability is developed. This model is helpful for evaluation of the total global consumer's risk as a combination of particular global risks of any number of the components under control. Evaluation of the total specific risk as a combination of the particular specific risks is also possible based on this model.

Analysis of such risks in customs control of completely denatured alcohols is detailed as a case study.

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Figure captions

Fig. 1. Venn diagram of the considered events. Events C_1 and C_2 , when test results for components 1 and 2, respectively, are in their acceptance intervals, and events B_1 and B_2 , when the true contents of components 1 and 2 are not actually within their tolerance intervals, are shown by ellipses. Other events of interest are indicated as intersections of these ellipses: $C_1 \cap C_2$ — test results for both components 1 and 2 being in their acceptance intervals simultaneously; $C_1 \cap B_1$ and $C_2 \cap B_2$ — test results for component 1 or 2, respectively, being in its acceptance interval, whereas corresponding true contents are not actually within their tolerance interval; $C_1 \cap C_2 \cap B_1$ and $C_1 \cap C_2 \cap B_2$ — test results for both components 1 and 2

being in their acceptance intervals simultaneously, when the true content of component 1 or 2, respectively, is not actually within its tolerance interval; $C_1 \cap C_2 \cap B_1 \cap B_2$ – test results for both components 1 and 2 being in their acceptance intervals, when none of the true contents of the components are within their tolerance intervals.

Fig. 2. Particular global customs risks R_{ci} at control of IPA and MEK concentrations. Curve 1 is for IPA (i = 1), and curve 2 – for MEK (i = 2); μ_i is the mean of the global distribution of true values of the denaturant concentrations c_1 and c_2 in CDA batches. The risk values at $\mu_1 = \mu_2 = 3.15 \text{ L} \cdot \text{hL}^{-1}$ are indicated by dotted lines 3 and 4 for IPA, and dotted lines 5 and 6 for MEK.

Fig. 3. Particular global customs risk R_{c3} at control of DB concentrations. μ_3 is the mean of the global distribution of true DB concentrations c_3 in CDA batches. The risk value at $\mu_3 = 1.10$ g·hL⁻¹ is indicated by dotted lines 2 and 3.

Fig. 4. Particular specific customs risk R_{ci}^* at control of IPA and MEK concentrations. Line 1 is for IPA (i = 1), and line 2 – for MEK (i = 2); c_{im} is the measurement result value. Dotted lines 3 and 4 mark an example for IPA, 5 and 6 – for MEK.

Fig. 5. Particular specific customs risk R_{c3}^* at control of DB concentrations. Line 1 is the dependence of the risk on the concentration measurement result c_{3m} . Dotted lines 2 and 3 indicate an example.

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