Analysis of Flow Meters Calibration

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Abstract

Calibration data of flow meters are often reported using the so called calibration coefficient, i.e. the ratio between the reference flow rate (or equivalent quantity, e.g. airspeed, accumulated volume etc.) and the corresponding quantity as indicated by the instrument to be calibrated. The main reasons for this choice are twofold:

- First of all, this approach is very practical for the end user of the instrument, who can get the corrected flow rate by simple multiplication of the readout times the coefficient;
- Second, this representation allows to highlight the non-linearities of the instrument, which usually show up in the lower end of the range and might be hidden by a direct representation.

On the other hand, this approach makes the uncertainty evaluation of the resulting data more complex, because of the correlation between the regression data.

In this paper, we will perform an analysis according to such traditional approach and also according to an alternate “direct” approach, i.e., considering the reference flow rate as the dependent variable, instead of the calibration coefficient. In both cases the regression will be performed by a specific software for calibration curves (CCC software, developed at INRIM in the framework of the EMRP NEW04 Project). Fig. 1 shows, as a case example, the scatter plot and the relevant regression curve for the two data representations.

A procedural approach for correctly performing regression and uncertainty evaluation will be derived for both methods, and the results obtained will be compared.

Specific attention will be devoted to the non-linear response region of the instrument range, since this part of the range is the more delicate and often the one where the instrument is used for a significant fraction of its operational life.

1 Introduction

Flow measurement has long been an important sector in metrology, due to its economic and scientific implications; on the one hand, huge amounts of energy-carrying fluids (like e.g. natural gas, oil, liquefied natural gas…..) are exchanged daily worldwide, driving a large activity in fiscal measuring; on the other hand, fluids (both liquids and gases) are the medium within which essentially every activity takes place, and the medium carrying all kinds of substance to the most various utilizations (e.g. oxygen in blood, pollutants in the atmosphere, reactants in pipes, etc.). It is therefore no wonder that the measurement of fluid flow, in its various aspects (e.g. anemometry, hydrometry, flow rate…..) has such a large field of applications.

The instruments for fluid flow measurement are mostly based on some physical phenomenon associated to the flow itself (e.g. mechanical action for vane anemometers or turbine gas meters, heat exchange for hot wire anemometers or thermal mass flow meters, etc.), therefore it is necessary to determine a functional relationship between the actual flow stimulating the sensor and the indication of the latter. This operation is the instrument calibration, and, in addition to the gathering of experimental points, it requires a mathematical treatment of such data.

The final form in which the calibration data are reported is often the so called calibration coefficient, i.e. the ratio between the reference value quantity of interest (e.g. flow rate, airspeed, accumulated volume etc.) and the output of the instrument to be calibrated. The main reasons for this choice are twofold:

- First of all, this approach is very practical for the end user of the instrument, who can get the corrected flow rate by simple multiplication of the readout times the coefficient;
- Second, this representation allows to highlight the non-linearities of the instrument, which usually show up in the lower end of the range and might be hidden by a direct representation.

This approach is particularly appreciated in the flow measurement community since most of the instruments commonly used are strongly nonlinear at least in a part of their range, and usually they are utilized throughout such range (and possibly a little beyond it). On the other hand, this approach makes the uncertainty evaluation of the resulting data more complex, because of the correlation between the regression data.

In order to deepen the understanding of the present approach and to explore alternate routes, in this paper we will perform the analysis of available calibration data either according to the previously described “standard” approach or to an alternate one based on a more straightforward treatment, i.e., considering the reference flow rate as the dependent variable, instead of the calibration coefficient. In both cases the regression will be performed by a specific software for calibration curves (CCC software, developed at INRIM in the framework of the EMRP NEW04 Project), which will be shortly described.

A procedural approach for correctly performing regression and uncertainty evaluation will be derived for both methods, and the results obtained will be compared.

Specific attention will be devoted to the non-linear response region of the instrument range, since this part of the range is the more delicate and often the one where the instrument is used for a large part of its operational life.

2 Data Gathering and Preparation

The data analyzed in the present paper refer to the internal calibration of three Mass Flow Controllers (MFCs), owned by INRIM, intended to be applied to the testing of online production of gas mixtures. The uncertainty associated with the measurements performed by them is therefore very important since it will have a direct repercussion on the uncertainty of the produced mixtures, which are expected to provide better qualitative results than the corresponding mixtures statically produced via the gravimetric method.

The MFCs were calibrated against the INRIM MICROGAS test rig, developed and validated at INRIM in the '90s [1],[2],[3]; the test rig is able to provide flow rates in the dynamic range spanning from 0.1 SCCM (Standard Cubic Centimetre per Minute) to 1500 SCCM, with an uncertainty as low as 0.03 % of the flow rate from 1 SCCM upwards. Calibrations were performed according to the standard procedure [4] and using 5.5 Nitrogen (i.e. Nitrogen with a purity of 99.9995 % or better) as a working fluid.

For each instrument, seven calibration points, corresponding to 5%, 10%, 17.5%, 30%, 45%, 70% and 100% of the full scale range (FSR) respectively, were measured, each point being repeated three times to check the repeatability of the instrument and that of its interaction with the test rig.

Data collected at the test rig were reduced using the standard INRIM method, based on the balance of mass between the beginning and the end of measurement, which allows to reduce the effect of the dead volume. A pair of data sets was built for each tested MFC.
According to the alternate approach, one data set shows the flow rate values Q_i indicated by the DUT (Device Under Test) as the x values (independent variable), while the reference flow rate values Q_r provided by the reference test rig after corrections as the y values (dependent variable). Such values were obtained by the balance of the mass method, which allows a more precise evaluation of the actual amount of gas actually supplied by the test rig.
On the other hand, according to the more common approach, the other data set has the same x values as those described above, but the y values are the calibration coefficient values K of the instrument, $K = Q_r/Q_i$.

The uncertainty associated to the x values is, in both cases, computed taking into account two main contributions, namely the resolution and the repeatability uncertainty of the DUT. The former contribution was evaluated by assuming a rectangular distribution with a width equal to one instrument digit, while the latter was evaluated based on the standard deviation of the repeated measurements performed during the set acquisition interval. Since the x values are independent from each other, the associated covariance matrix is simply a diagonal matrix with the squared standard uncertainty of the Q_i value as its diagonal terms.

Regarding the evaluation of the uncertainty associated to the Q_r values, the following contributions were considered:

a) the uncertainty on the thermodynamic state of the gas (temperature and pressure); for both these influence quantities, the calibration, the resolution and the drift uncertainty contribution of the used instruments were considered, together with a repeatability contribution evaluated by assuming a rectangular distribution on the interval defined by the maximum and minimum measured values;
b) the uncertainty on the measured volume (difference between the final and the initial volumes) evaluated by keeping into account the uncertainty of the interferometer and the possibility of small leaks from the seal;
c) the uncertainty on the dead (initial) volume of the test system; this consists of a rough and conservative value, however it does not influence greatly the output uncertainty since it is associated to a very small sensitivity coefficient;
d) the uncertainties on the physical constants used in the analysis (gas constant and molar mass of the gas) can be shown to provide a negligible effect on the final uncertainty.

Finally, the (relative) uncertainty associated to the calibration coefficients K was evaluated by summing quadratically the uncertainties associated to Q_i and Q_r and a repeatability term obtained from the dispersion of the coefficients.

In both the approaches, the y values are considered to be independent from each other; of course, this is a stronger simplifying assumption than in the case of the x values, but it is considered acceptable at this stage. The associated covariance matrix is therefore a diagonal matrix with the squared standard uncertainty of the y values as its diagonal terms.

3 Data analysis

Once the data sets were ready, together with relevant uncertainty matrices, they were used to obtain the corresponding analysis curve by means of a weighted total squares technique [5]. The regression was performed by using the CCC software, developed at INRIM, in the framework of EMRP Project NEW04 [6], for the determination of calibration and analysis curves based on experimental data. The software allows to fit the data through fractional polynomials; the number and the degree of the monomial terms forming the fitting model can be chosen by the user; available degrees include integer values from -5 to +5 with the addition of power values -½ and +½. The software provides estimates of the polynomial coefficients and an associated covariance matrix.

As described in Par. 2, the applied weighted total squares procedure allows to take into account the uncertainties in both the dependent and the independent variable; future developments will include also the covariances among the values of each variable.. To all the data sets, polynomials of several degrees were fitted; the optimum choice was taken as the one leading to a (normalized) chi-squared (χ^2) value lower than 5, while showing the lowest possible number of monomial components.

3.1 Calibration Curves Determination

For each MFC, two sets of data were prepared as described in Par. 2. From each set, a fitting curve relating the readings of the instrument on the *x* axis to the chosen *y* values (reference values of the flow rate Q_r or calibration coefficient *K* values) was determined. Notice that the curve thus determined is, in mathematical terms, an analysis curve, although it is usually referred to as calibration curve.

Fig. 1 reports, as an example, the plot of the couple of curves obtained for Case 2 (500 SCCM FSR instrument), while Table 1 reports the mathematical formulation of the calibration curves obtained for all cases.

It can be observed in Fig. 1 that the appearance of the calibration curve for Case 2a is that of a simple straight line, while the elaboration of Case 2b clearly highlights, already from a visual standpoint, the strong non linearity of the instrument response, particularly at the lower flow rates.

On the other hand, the equations in Table 1 show that, also for Case 2a, the actual behavior is nonlinear. In fact, when trying to fit the data with a simple straight line, the value of the normalized χ^2 was found to be very high, thus indicating an inappropriate fit.

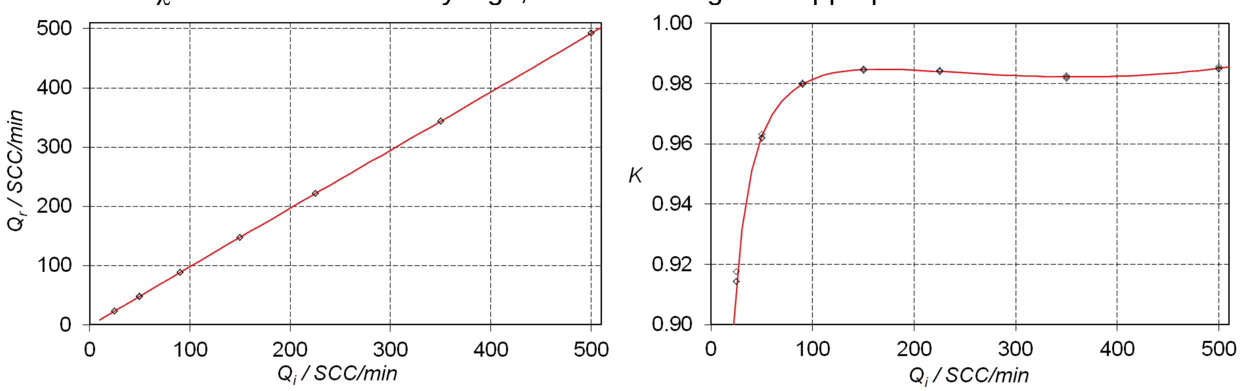

Fig.1. Calibration Curves for Case 2. Left: case 2a, calibration based on the reference value Qr; right, case 2b, calibration based on the coefficient *K*.

Case:	MFC Range	Equation of the fit	Normalized χ^2
	(SCCM)		(-)
1a	200	$-0.7099 + 1.05168\,x - 0.0006\,x^2 + 1.6419 \cdot 10^{-6}\,x^3$	2.1
1b		$-\frac{0.7092}{x} + 1.05178 - 0.0006\,x + 1.6498 \cdot 10^{-6}\,x^2$	1.1
2a	500	$-2.5426 + 1.02065\,x - 0.00015\,x^2 + 1.88 \cdot 10^{-7}\,x^3$	4.0
2b		$-\frac{2.5297}{x} + 1.02022 - 0.00015\,x + 1.8425 \cdot 10^{-7}\,x^2$	1.2
3a	2000	$-29.466 + 2.3636\,\sqrt{x} + 0.89816\,x + 5.4215 \cdot 10^{-5}\,x^2 + -7.4646 \cdot 10^{-12}\,x^4$	4.5
3b		$-\frac{30.325}{x} + \frac{2.5029}{\sqrt{x}} + 0.89259 + 5.6511 \cdot 10^{-5}\,x + -7.732 \cdot 10^{-12}\,x^3$	2.6

Table 1. Calibration curve equations for the various test cases.

It can be observed, when comparing the equation couples, that in each case the version “b” fit includes the same number of terms as the version “a” set, and that all terms are one degree lower than the corresponding ones. In addition, the monomial coefficients are very similar for corresponding terms, although not exactly the same. This is consistent with the fact that in case “a” the dependent variable is *y*, whereas in the case “b” it is *y*/*x*.
Another point that is worth observing is the value of the normalized χ^2 parameter, which is an estimate of the goodness of the fit. The first observation is that, in all cases, the “b” elaboration allows a better result than the “a” elaboration. Also, both elaborations of case 3 did not allow to reach values as low as the ones obtained for the cases 1 and 2, although within the required limit.

3.2 Comparison of the Results

In order to check the difference between the performances of the two methods, a new data set, with nominal flow rates different from those used in the calibration, was measured. The new data points, in both forms, are reported graphically in Fig. 2, together with the calibration points.

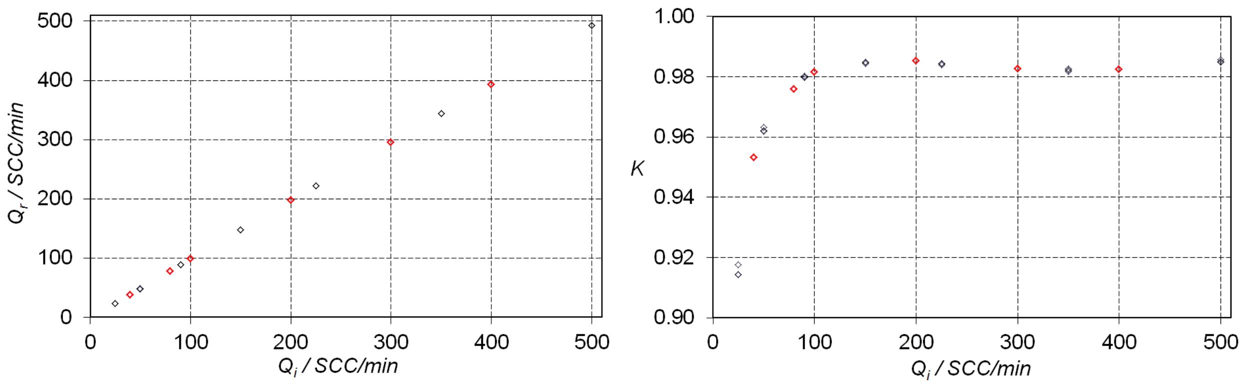

Fig.2. Supplementary data set, Case 2. Left: case 2a; right, case 2b. Calibration points in black, supplementary points in red.

These points were compared to the corresponding point reconstructed from the calibration curves at the same measured x position.
The uncertainty associated to the reconstructed flow rates is computed by means of the law of propagation of uncertainty applied to the fitting model, by which the uncertainty associated to the analysis curve coefficients (provided by the CCC software together with all other mathematical parameters connected to the elaboration of the data) and the uncertainty of the new Q_i values were propagated, while the uncertainty associated to the measured points was evaluated as described earlier.

The (absolute) expanded uncertainties associated to the measured values (Q_{meas}) and to the two reconstructed values (Q_{rec}) were thus computed for all the six test points. Using these values, the compatibility parameter $E_n = \frac{|Q_{meas} - Q_{rec}|}{\sqrt{U^2(Q_{meas}) + U^2(Q_{rec})}}$ was also computed; the results are presented in Table 2

Q_{meas}	Q_{rec}, 2a	Q_{rec}, 2b	$U(Q_{meas})$	$U(Q_{rec})$, 2a	$U(Q_{rec})$, 2b	E_n, 2a	E_n, 2b
38.125	38.050	38.050	0.0103	0.0918	0.0952	0.809	0.786
78.067	78.219	78.212	0.0190	0.1290	0.1513	1.162	0.947
98.158	98.165	98.156	0.0265	0.1533	0.1882	0.046	0.006
197.049	196.903	196.903	0.0446	0.3351	0.4770	0.432	0.304
294.816	294.803	294.819	0.0650	0.6504	0.9881	0.021	0.003
393.009	392.981	392.998	0.0853	1.1261	1.7777	0.025	0.006

Table 2. Comparison of results.

It can be observed that, for all test cases, the compatibility parameter is larger for the case "a" reconstruction, thus indicating a lesser compatibility between the reconstructed and the actually measured data. In particular, one point (corresponding to the 80 SCCM nominal flow rate) has a compatibility index larger than one, highlighting a non-compatibility between the two data sets. It has to be noticed that the coefficient is only slightly larger than one, and sufficiently so to keep

the point in the so-called "grey zone", but the fact remains. At the same position, the case "b" reconstruction allows to obtain a result compatible with the measured data (although not by much).
Another interesting observation is that the point just discussed lies in the region of maximum nonlinearity of response of the instrument. This underlines once more that special care must be taken in treating such zones, and confirms quantitatively the hypothesis that the "b" type of reconstruction, based on the calibration coefficient analysis, allows a better treatment of the nonlinear response of flow measuring instruments.
The overall analysis of the compatibility parameters also shows that the most delicate part of the instruments' range is actually its lower part, where in all cases quite high values of E_n are obtained.
A final observation is that, in any case, the determination of the analysis curves by the CCC software allowed a satisfactory result with both approaches, although, as shown, the calibration coefficient approach still has some advantage points.

4 Conclusion

In this paper we compared two different approaches to the determination of calibration curves of flow meters. The comparison on a rigorous mathematical standpoint, based on the optimal polynomial fit for the chosen approach, was allowed by the use of a novel software tool developed at INRIM in the framework of the EU EMRP Programme.
The conclusion of the comparison is that the "traditional" approach of using calibration coefficients is actually justified, despite the increased complexity of the uncertainty analysis, since it allows a better understanding of the instrument nonlinearities.
On the other hand, the advantage of this approach is greatly reduced by the use of sophisticated mathematical tools which allow to highlight and compute the nonlinear behavior of the instruments also with the more direct approach.
Indeed, it was shown that, through use of such mathematical tools, the full behavior of the instrument can be expressed and nonlinearities detected. The overall quantitative advantage in reconstruction of the traditional approach still exists but is not very large when optimal parameters are computed for both approaches.

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