The third-millennium International System of Units

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Summary. — The International System of Units, SI, in use since 1946, was formally established in October 1960 by the eleventh Conférence Générale des Poids et Mesures, CGPM, with its resolution 12. In the past years, several changes have been made to the system. The 26th Conference, on 16th November 2018, adopted a revised SI, due to come into force on 20 May 2019. This revision was by far the most radical in the history of the SI. In this paper, I review the system from its origin to the present time, discuss the needs that suggested, and the conditions that allowed such an epochal change, and present the SI of the third millennium.

EVERY expression of a Quantity consists of two factors or components. One of these is the name of a certain known quantity of the same kind as the quantity to be expressed, which is taken as a standard of reference. The other component is the number of times the standard is to be taken in order to make up the required quantity. The standard quantity is technically called the Unit, and the number is called the Numerical Value of the quantity.

James Clerk Maxwell

1. – Introduction

The sentence above, placed at the very beginning of Maxwell's Treatise on Electricity and Magnetism of 1873 [1], is customarily written in mathematical terms as

$$Q = \{Q\}[Q], \tag{1}$$

where the quantity Q is expressed as $\{Q\}$ (the numerical value of the quantity) times the suitable unit $[Q]$. This notation is adopted throughout the paper. The seemingly innocuous expression (1), which I suspect Maxwell wrote simply to clarify the role of a unit, led to the so-called quantity calculus [2], and to a series of subtle distinctions between quantity, value of a quantity and numerical value of a quantity [3]. The debate on whether a unit should be considered a quantity or a quantity value is still alive in contemporary times and relevant to the revision of the International System of Units [4,5]. Nonetheless, I consider this as a minor issue and I will no longer deal with it in this review.

2. – Historical overview

Although this paper is a review of the new International System of Units, it is appropriate to carry out a brief historical overview. I do not intend to start from the very beginning of the story, as this would be far beyond the scope. I will just have a look at the roots of the modern systems of units.

2˙1. *Precursors: the decimal system.* – The *États-généraux* (Estates General), solemnly opened in Versailles, near Paris, on the 5th May 1789, had a special importance, both for the great history of mankind and for our small history of weights and measures. The Estates General had been established in 1302 by the King Philippe le

Beau as an extraordinary assembly aimed at giving a semblance of democracy to the king's decisions so as to legitimate them to the eyes of the population. The Estates General, a sort of *Ancien Régime* precursor of the National Assembly, was the meeting of the representatives of the three classes, or *Estates*, of the realm, the clergy, the nobility and the commoners, that is everyone else, and had essentially consultative functions, predominantly on fiscal matters. The Estates General was also the occasion to present to the King the *Cahiers de doléances*, lists of grievances drawn up by each of the three Estates. The 1789 Estates General were the first to be summoned again since 1614, and the *Cahiers* are particularly relevant to the history of the systems of units.

Il y aura dans toute l'étendue du royaume uniformité de poids et de mesure, et seront les poids et mesures portés au titre et terriers des seigneurs, réduits aux poids et mesures adoptés par les Etats généraux(1).

This is just an example of a very large number of requests found in the 1789 *Cahiers*. The requests were far from unjustified. Towards the end of the 18th century the situation of units was chaotic, not only among the different countries, but even within the same country. For example, in 1788 in France about 2000 units are reported, among which 200 different *livres*! [6]

The French Revolution impacted deeply on the situation mentioned above. The first legal basis to a decimal system was the law of 18 Germinal, Year III (7 April 1795). Article 5 of the law defined five units, for lenght, surface (land) area, solid and liquid volume, and mass.

The history of the path that led to the internationalization of the units, sanctioned by the Metre Convention of 1875 [7], is beyond the scope of this paper. For an exhaustive account of that fascinating process, see [6, 8].

3. – The International System of Units — from birth to 2018

3·1. *Birth.* – The official birth of the International System of Units dates from October 1960, when the eleventh Conférence Générale des Poids et Mesures, CGPM, in its Resolution 12 [9] stated

> The 11th Conférence Générale des Poids et Mesures (CGPM),
>
> *considering*
>
> – Resolution 6 of the 10th CGPM, by which it adopted six base units on which to establish a practical system of measurement for international use:

length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	degree kelvin	K
luminous intensity	candela	cd

(1) There shall be over the whole kingdom uniformity of weights and measures, and the weights and measures adopted in the registers of the lords shall be reduced to the weights and measures adopted by the Estates General [Author's free translation].

– Resolution 3 adopted by the Comité International des Poids et Mesures (CIPM) in 1956,

– the recommendation adopted by the CIPM in 1958 concerning an abbreviation for the name of the system, and prefixes to form multiples and submultiples of the units,

decides

1) the system founded on the six base units above is called the "Systéme International d'Unités";

2) the international abbreviation of the name of the system is SI;

3) names of multiples and submultiples of the units are formed by means of the following prefixes:

...

4) the units listed below are used in the system, without excluding others which might be added later.

...

As it is clear from the text, the Resolution was more a recognition of an existing state of affairs than the formalization of a natural birth. Actually, the system had been outlined since the 41st meeting of the CIPM in 1946, when Resolution 2 defined mechanical and electrical units [10] essentially in the form we know. It is worth noting that at that time the CIPM counted among its members two Nobel laureates, Louis de Broglie and Manne Siegbahn (although they both could not attend that session). Subsequently, the 9th General Conference [11] in 1948 decided to establish the International System, whose first six base units of length, mass, time, electric current, thermodynamic temperature and luminous intensity were defined by Resolution 6 of the 10th General Conference in 1954 [12]. Then came the 1960 formalization. This date is conventionally considered to be the official birth of the SI. The unit of amount of substance, the mole, was added as the seventh base unit in 1971 with Resolution 3 of the 14th General Conference [13].

3˙2. *Structure*. – The SI until 2018 is coded in the official SI Brochure [14], 8th edition. The structure of the base unit definitions in the SI reflects its historical evolution. There are three types of definitions:

– Those based on an artefact standard.

– Those based on a natural standard.

– Those based on a fundamental constant.

The only and last member of the first category is the unit of mass. It is defined as

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

In mathematical terms, this definition can be formalized as

$$\mathrm{kg} = m(\mathfrak{K}), \tag{2}$$

where $\mathfrak{K}$ denotes the international prototype.

To the second category belongs the definition of the time unit, the second, symbol s:

> *The second is the duration of* 9 192 631 770 *periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cæsium* 133 *atom.*

Therefore, from eq. (1)

$$\mathrm{s} = 9\,192\,631\,770 \frac{1}{\Delta\nu_{\mathrm{Cs}}}\,. \tag{3}$$

The definition implies that $\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770\,\mathrm{Hz}$ exactly. So, the definition of the unit defines at the same time the constant upon which it is based. As an aside consideration, however rich of implications that will be discussed later on, the hyperfine splitting in the cæsium ground state is far from being a fundamental constant, being rather a natural standard. Note that it is not a Lorentz invariant.

In the same category there is the definition of the temperature unit, the kelvin, symbol K:

> *The kelvin, unit of thermodynamic temperature, is the fraction* 1/273.16 *of the thermodynamic temperature of the triple point of water.*

This is expressed as

$$\mathrm{K} = \frac{1}{273.16} T_{\mathrm{tpw}}\,. \tag{4}$$

In the case of the temperature unit as well, the definition of the unit defines implicitly the natural standard on which it is based, the triple point of water.

$$T_{\mathrm{tpw}} = 273.16\,\mathrm{K}. \tag{5}$$

Among the definitions belonging to the third category there is that of the length unit, the metre, symbol m:

> *The metre is the length of the path travelled by light in vacuum during a time interval of* 1/299 792 458 *of a second.*

$$\mathrm{m} = \frac{\mathrm{s}}{299\,792\,458} c_0, \tag{6}$$

or

$$c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}. \tag{7}$$

This definition is not self-consistent, in that it implies that the second be defined previously.

Further on, the ampere, unit of electric current, symbol A, is defined as follows:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed* 1 *metre apart in vacuum, would produce between these conductors a force equal to* 2×10^{-7} *newton per metre of length.*

Writing the force in terms of the unit of current A and the magnetic constant μ_0, $F = \mu_0 \mathrm{A}^2/2\pi = 2 \times 10^{-7}\mathrm{N}$, one obtains

$$\mathrm{A} = \mathrm{N}^{1/2} \left(\frac{\mu_0}{4\pi \times 10^{-7}} \right)^{-1/2} . \tag{8}$$

This implies that $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N\,A^{-2}}$ exactly, and explains the reason for the value of the magnetic constant.

Note that for the definitions based on fundamental constants, these possess exact values, either because they were fixed at some time, as the speed of light in vacuum c_0, or because they were determined by construction, as the magnetic constant μ_0. They are both Lorentz invariant.

Also the mole, unit of amount of substance $n(\mathrm{X})$, belongs to those units defined in terms of a fundamental constant, the molar mass of carbon 12 $M(^{12}\mathrm{C})$:

> 1) *The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in* 0.012 *kilogram of carbon* 12*; its symbol is* "mol"*.*
>
> 2) *When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.*
>
> It follows that the molar mass of carbon 12 is exactly 12 grams per mole, $M(^{12}\mathrm{C}) = 12\,\mathrm{g\,mol^{-1}}$.
>
> In 1980 the CIPM approved the report of the CCU (1980) which specified that
>
> In this definition, it is understood that unbound atoms of carbon 12, at rest and in their ground state, are referred to.
>
> (From [14], page 115).

To encompass this definition in the same formal scheme as the previous definitions, one writes

$$\mathrm{mol}(\mathrm{X}) = \frac{0.012\,\mathrm{kg}}{m(^{12}\mathrm{C})} n_{\mathrm{e}}(\mathrm{X}), \tag{9}$$

where $n_{\mathrm{e}}(\mathrm{X})$ is the amount of substance of one elementary entity and the dimensionless factor $0.012\,\mathrm{kg}/m(^{12}\mathrm{C})$ is the number of entities specified in the definition, that is, the number of $^{12}\mathrm{C}$ atoms in 12 g of carbon 12. The molar mass being defined as the ratio of the mass of a sample to its amount of substance, using expression (9) with $(^{12}\mathrm{C})$ yields

$$\frac{m(^{12}\mathrm{C})}{n_{\mathrm{e}}(^{12}\mathrm{C})} = M(^{12}\mathrm{C}) = 0.012\,\mathrm{kg\,mol^{-1}}, \tag{10}$$

as stated in the definition.

The universal proportionality constant relating in any sample the number $N(\mathrm{X})$ of elementary entities X and the corresponding amount of substance $n(\mathrm{X})$, also denoted the molar number of entities [15], is the well-known Avogadro constant N_{A}. From definition (9) of the mole it follows that one mole of substance contains N_{A} entities of that substance. Now, since the number of entities is dimensionless, it follows also that in the SI N_{A} has unit mol^{-1}. This statement holds before and after the 2018 revision of the system. What changes, as we will see, is that in the previous system N_{A} had to be determined experimentally, whereas in the current system is assigned an exact value, at the expenses of the molar mass of carbon 12.

The mole, the most recent of the seven base units of the SI, was adopted in 1971 by the 14th CGPM [13] in an attempt to attract the world of chemistry within the structures and the language of international metrology. Judging from the present situation, the attempt was absolutely successful: the *Consultative Committee for Amount of Substance: Metrology in Chemistry and Biology (CCQM)* is by far the largest among the CIPM Consultative Committees. All the great National Metrology Institutes (NMIs) have a Department devoted to this topic, which in many cases is at least as large as those devoted to physical metrology.

The last base unit (in order of listing, not in a chronological sense) concerns one of the so-called physiological quantities, the luminous intensity. This is the radiant intensity (having unit watt per steradian) weighted by a luminous efficiency function that accounts for the average spectral sensitivity of the human eye at the various wavelengths. The unit of luminous intensity is the candela, symbol cd, defined as

> *The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency* 540×10^{12} *hertz and that has a radiant intensity in that direction of* $1/683$ *watt per steradian.*

As for other units, this definition also defines the value of the constant on which the definition is based, the so-called spectral luminous efficacy, symbol K_{cd}, of the monochromatic radiation of frequency 540×10^{12} hertz to be $K_{\mathrm{cd}} = 683\,\mathrm{lm\,W^{-1}} = 683\,\mathrm{cd\,sr\,W^{-1}}$ or $K_{\mathrm{cd}} = 683\,\mathrm{cd\,sr\,kg^{-1}\,m^{-2}\,s^{3}}$.

In mathematical terms

$$\mathrm{cd} = \frac{K_{\mathrm{cd}}}{683}\,\mathrm{kg\,m^2\,s^{-3}\,sr^{-1}}\,. \tag{11}$$

The base quantities behind the base units are by convention regarded as independent. The SI base units are by no means independent. Figure 1 depicts the relationships between the base units.

As previously mentioned, the definitions of the base units in the SI prior to 2018 reflect the historical evolution of the system. Therefore, their formats are different, depending on the kind of definition. As is well known, the definitions of some units, such as the second and the metre, were also changed over time. In addition, the derived units, which I will not discuss here, evolved accordingly. Ultimately, the system is not something static but has evolved during its first (almost) sixty years. For a concise history of the SI, see [16].

However, none of its modifications is comparable to the change occurred in 2018.

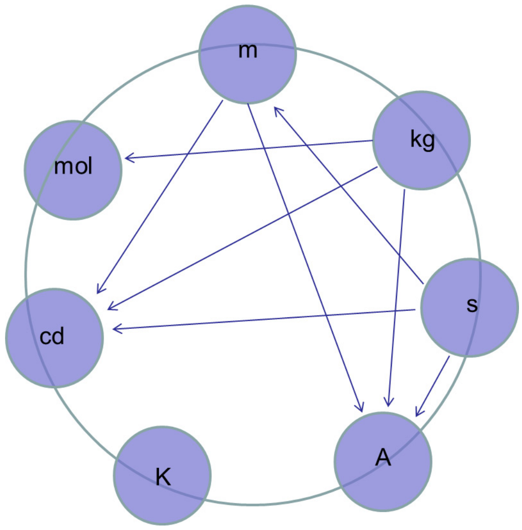

Fig. 1. – Relationships among the SI base units.

4. – Areas of improvement

In 2010, during the 20th meeting of the Consultative Committee for Units, CCU, the 50th birthday of the SI was celebrated with a simple ceremony (fig. 2). It is significant that at the same time the process towards a revised, better and hopefully more stable SI was already well under way.

Several areas of improvement are evident, both from the formal and, more important, from the substantial viewpoint.

On the formal side, the definitions appear widely different, as already mentioned. Although this feature does not impair the robustness and the wide applicability of the system, a better consistency in the format of the definitions would be desirable under many respects. The most evident drawbacks are that many of the definitions of the base units are either circular (base units are defined in terms of derived units), and (or) incomplete (they are defined in terms of another base unit). To the first category belongs the second, ultimately defined in term of the hertz, or the ampere, defined in terms of the derived unit newton. To the second, the metre, whose definition requires that the second be previously defined.

From the point of view of substance, the most important issue was by far the definition of the mass unit. Indeed, this was the main motivation for the proposal that revolutionized the system. A second issue was the situation of electrical units. A third problem was the uncertainty of many fundamental constants. The situation of the temperature unit was not critical but unsatisfactory, whereas the chemical community could be considered comparatively happy. A (unresolved) issue concerned dimensionless quantities, plane and solid angles in particular.

4˙1. *The kilogram.*

> *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram* [14].

Fig. 2. – Ian Mills and Claudine Thomas, CCU President and Executive Secretary, respectively, celebrate the 50th birthday of the SI (courtesy BIPM).

A picture of the international prototype, usually denoted as $\mathfrak{K}$, is given in fig. 3.

This definition dates back to the third CGPM [17] in 1901. Already in 1889 the first General Conference had sanctioned the international prototypes of the metre and the kilogram:

> The Conférence Générale des Poids et Mesures,
>
> *considering*
>
> – . . .
>
> – the equality in length of the international Metre and the equality in mass of the international Kilogram with the length of the Metre and the mass of the Kilogram kept in the Archives of France;
>
> – . . .
>
> *sanctions*
>
> A. As regards international prototypes:
>
> 1) The Prototype of the metre chosen by the CIPM. This prototype, at the temperature of melting ice, shall henceforth represent the metric unit of length.
>
> 2) The Prototype of the kilogram adopted by the CIPM. This prototype shall henceforth be considered as the unit of mass.
>
> . . . (see [18], p. 34, English translation by the BIPM).

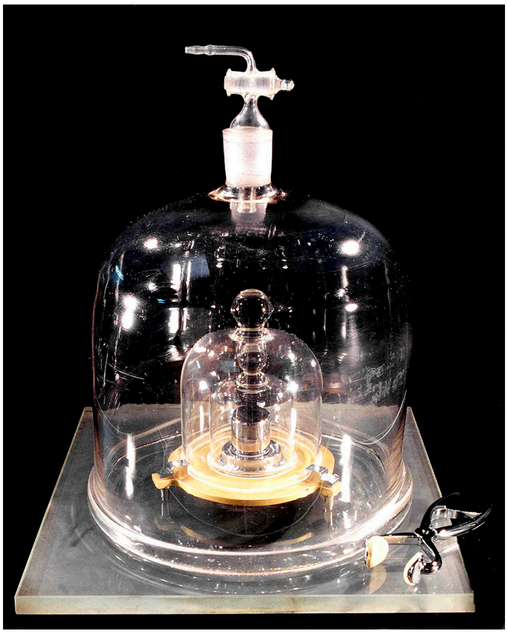

Fig. 3. – $\mathfrak{K}$, the international prototype of the kilogram (courtesy BIPM).

The definition of the metre was abrogated in 1960 by the 11th CGPM, while that of the kilogram remained unchanged until November 2018. As such it was at that time the oldest unchanged definition of the SI. It was also the last one by which a unit was defined in terms of an artefact.

There are at least two orders or reasons for the prolonged lack of innovation in the definition. On the one hand, the accuracies achievable in mass metrology have been traditionally more than adequate to the needs not only of commerce, but also of technology and science; therefore, little motivation was felt to change a definition that looked perfectly fit for purpose. On the other hand, even when, some decades ago, the drawbacks inherent in the definition began to emerge, thanks to the improvements in the balances and the subsequent reductions of uncertainties, the task of finding a better definition proved extremely challenging.

The unique feature of the unit of mass, that of being defined in terms of an artefact, has traditionally implied a number of consequences and misbeliefs. To mention just the most widespread among the latter category, it is commonly believed that mass metrology is a low-accuracy discipline, compared with, say, the metrology of electrical quantities. Yet, state-of-the-art mass comparators can appreciate 0.1 μg differences, at the kilogram level, with uncertainties in the range of 0.2 μg, that is to say, in relative terms, 1×10^{-10} and 2×10^{-10}, respectively. The national copies of the international prototype used to be calibrated by the BIPM with a standard uncertainty equal to 3 μg, or a relative standard uncertainty of 3×10^{-9}. I deliberately wrote "used to be", because in the last years before the re-definition of the kilogram the BIPM, due to an unexpected systematic error [19, 20], increased the uncertainty given above and realized that the usual value was perhaps a little bit optimistic.

By the way, these uncertainties are not too bad, compared to what can be achieved with other base quantities of the SI. They are certainly better than what thermometry can achieve, let alone photometry and radiometry, or chemistry, where concentrations are rarely determined to within some parts in 10^5.

As we will see further on, these uncertainties were challenging even for the redefinition of the kilogram.

One of the reasons for this good performance is that, by its own nature, realizing a definition based on an artefact is an elementary operation, in that the artefact *is* itself the realization. As such, the realization is unaffected by uncertainty, contrary to what occurs for other base units. The uncertainty inherent in the realization of the unit is the unavoidable, topmost contribution to the uncertainty of any measured value of the quantity corresponding to the unit itself, be it a value realized by a reference standard or a value measured at the shop-floor level.

The fact that using an artefact-based definition avoids the uncertainty due to the realization of the definition might look a good feature. However this seemingly desirable feature implies the heavy counterpart that nothing can be claimed about the time invariance of the unit. The time invariance cannot be taken as granted also for any other base unit. For example, the unit of length, the metre, is based on the speed of light in vacuum c_0, which has the exact value $c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}$. It cannot be claimed with certainty that this value is invariant in time and, as a matter of fact, theories exist, such as string theory, according to which it might change (for an early suggestion, see [21]), as well as the values of the other fundamental constants. However there is strong experimental evidence that if changes are occurring, they occur at a pace which is by no means a worry for the future centuries. For example, experiments, based on comparisons of atomic clocks, set on the drift of α, the fine structure constant, the constraint $(1/\alpha)\ (\mathrm{d}\alpha/\mathrm{d}t) = -0.20(20) \times 10^{-16}/\mathrm{yr}$ [22, 23]. For a recent review, see [24].

It can be concluded that fundamental constants are comfortably stable over a time interval which far exceeds the needs of any practical application, such as their use in a system of units.

The situation is completely different with an artefact such as the international prototype of the kilogram $\mathfrak{K}$ and its predecessor, the kilogramme des Archives, KA. The latter can be considered as the first standard of mass conceived upon rational considerations and intended to serve not only in trade, but also in science. It is really a product of the French Revolution, and epitomizes the ideal of a unique, universal unit, based on a constant of nature. Indeed, the kilogram was defined as the mass of a litre, or cubic decimetre, of distilled water at its melting temperature. This first definition of the kilogram was materialized by the kilogramme des Archives, delivered to the Archives de France, where it is still kept nowadays, in June 1799. KA is a platinum cylinder hot-forged from a platinum sponge, due to the impossibility of melting platinum at that time. The melting temperature of platinum is $2041.4\,\mathrm{K}$, or $1768.3\,^\circ\mathrm{C}$, and having a suitable material for a crucible in which to cast the melted platinum is a technical challenge that lasted for a long time.

The link of the definition to a constant of nature was not due to last, as the change in the reference conditions for water from the melting temperature to that of its maximum density (at the atmospheric pressure in both cases), *i.e.*, about $4\,^\circ\mathrm{C}$, had the effect that when KA was sanctioned, in December 1799, the kilogram was defined as the mass of KA itself, thus abandoning any reference to a constant of nature [25].

The kilogramme des Archives was a French affair. Its successor, the international prototype of the kilogram $\mathfrak{K}$, was the outcome of an international effort, as described below. Reference [8] was a precious source for many of the aspects discussed in this section.

Following the agreements of the Metre Convention, the company Johnson Matthey, UK, provided early in 1879 three cylinders to a special Committee charged with the task of producing the international prototype, by adjusting and polishing the cylinders. These were made of a new alloy of 90% platinum and 10% iridium, with a melting temperature even slightly higher (1790 °C) than that of platinum. This was but one of the enormous difficulties presented by the production of the new alloy [26, 27].

The three standards were compared with the kilogramme des Archives at the Paris Observatoire, and the prototype was selected as the one whose mass was closest to that of KA.

A further batch of 40 cylinders was commissioned to Johnson Matthey to manufacture further copies of the prototype and the national prototypes to be distributed to the countries signatories of the Metre Convention. Italy, as one of the first signatories, received in 1889 the prototypes n. 5 and n. 19, serving the former as the national prototype and the latter as its copy, or prototype of the second order. More recently, the Istituto di metrologia "Gustavo Colonnetti", IMGC, subsequently merged with the Istituto elettrotecnico nazionale "Galileo Ferraris", IEN, to become, since 1 January 2006, the Istituto Nazionale di Ricerca Metrologica, INRiM, ordered to the BIPM the prototypes n. 62 in 1974 and 76 in 1993. The former is the national standard of mass, according to the law [28], whereas n. 5 remains the national prototype. This can be viewed as a further anomaly of our national system. Happily, with the new SI, I will no longer need to explain the situation to my incredulous foreign colleagues.

The number of Pt-Ir kilograms manufactured so far is 114, including the international prototype [29]. Most of them are of course national prototypes, some are kept at the BIPM either as copies of $\mathfrak{K}$ (*témoins*, in the number of six) or as working standards. One, allocated to Argentina in 1939, was lost in 1986 during the travel from the BIPM back to Argentina. For a list of the prototypes, up to n. 84, see [26] and [30].

The tolerance on the national prototypes is equal to ±1 mg. Along their history, the national prototypes have been subjected to three periodic verifications (official term for re-calibration) at the BIPM, where they have been compared, more or less directly, with the international prototype $\mathfrak{K}$.

The first verification took place from 1899 to 1911 and $\mathfrak{K}$ was not involved. The second began in 1939 and was re-started in 1947 after a long stop imposed by the second World War, to be finished in 1954. The third started in 1988 and ended in 1992.

4·1.1. Evidence for the instability of $\mathfrak{K}$. On the occasion of the third verification it became evident that the prototypes were unstable. Of course, the usage of $\mathfrak{K}$ was kept to a minimum, to minimize the risk of damage and the wear induced, for example, by the friction of the base of the standard against the plate of the balance during weighings, when placing and removing it from the plate itself. This manoeuvre has always been performed by remote operation on the first BIPM balance, and in an automatic way subsequently. The point is that a comparison implies a great number of exchanges on the balance pan. The mass decrease due to wear is irreversible, but is compensated by the (in principle reversible) effect of accumulation of dirt both during storage and manipulation. It is worth noting that the prototypes are simply kept under two glass bells, three for $\mathfrak{K}$, which are not air-tight.

The characteristic of the SI unit of mass of being an artefact was perceived as a weakness since the official birth of the SI. During his reply to the opening address, by the French Minister of Foreign Affairs, to the 11th General Conference in 1960, the same that sanctioned the International System, André-Louis Danjon, the French astronomer president of the International Committee for Weights and Measures, CIPM, said:

> *Seule des trois grandeurs fondamentales de la mécanique, la masse conservera un étalon artificiel, le Kilogramme en platine iridié du Pavillon de Breteuil; or, sans invoquer l'un de ces cataclysmes dont on nous fait un épouvantail et qui pourrait fort bien le volatiliser, il faut bien avouer que son invariabilité tient un peu du miracle. En pratique, on ne l'utilise que rarement, de peur de l'altérer. Il y a là une faiblesse du Système Métrique à laquelle les métrologistes devront tôt ou tard porter remède*(2) [31].

During the previous verifications, especially during the long pause imposed on the second one by the World War, studies were carried out at the BIPM on how to best clean the prototypes. These studies eventually were standardized in what was called *cleaning-washing* procedure [32] (fig. 4). This procedure is highly effective in removing impurities from the surface, having the main drawback of being operator-dependent. This is a minor drawback, as for many years only one person at a time was in charge at the BIPM with this task, thus ensuring a good reproducibility of the procedure.

The third verification differed from the previous ones in some important respects. First, the balance used for the comparisons, the NBS-2, donated to the BIPM by the National Bureau of Standards, NBS, now NIST (USA), had a standard deviation of about 1 μg, far better than the 10 μg of the Rueprecht balance previously used. Second, the effects of cleaning-washing on the prototypes and the accumulation of contaminants were systematically studied and better understood, also thanks to the better performance of the balance.

The prototypes were first weighed as received, then cleaned and washed, then weighed again prior to the calibration. This measure gave an idea of the amount of contamination accumulated by each prototype. Two BIPM prototypes were never cleaned-washed, and $\mathfrak{K}$ (as well as other official copies) was compared against them before and after cleaning. A previously undetected phenomenon appeared, affecting all the prototypes, including $\mathfrak{K}$. This phenomenon consists in a rapid increase of mass, of the order of about 1 μg per month, in the weeks and months immediately following the cleaning and washing procedure. This rate lasts for six months to one year, to become about 1 μg per year subsequently, as it had been known for a long time. This behaviour made it evident that the 1901 definition, referring generically to the mass of the international prototype, was insufficient, since that mass was rapidly changing during the months when the prototype was used for the dissemination of the unit. Changing the definition in time for the third verification, already under way, would have been impossible, as such a change would have needed a sanction by the CGPM. Therefore, the CIPM, rather than changing the definition of the SI unit of mass, decided to complement it with a better specification of what is to be intended for the mass of the international prototype. Terry J. Quinn, at that time Director of the BIPM, suggested to the CIPM to interpret the mass of $\mathfrak{K}$ as the mass it has immediately after the cleaning and washing procedure, and that the value of the mass at the moment of using $\mathfrak{K}$ in the subsequent period be extrapolated. The CIPM accepted [33]. This decision is also reflected in the SI brochure, appendix 2,

(2) Unique among the three fundamental mechanical quantities, mass will continue to have an artefact standard, the platinum-iridium kilogram of the Pavillon de Breteuil; now, with no need to imagine one of those scaring catastrophes which could very easily volatilize it, it must be admitted that its invariability is something of a miracle. In practice, it is rarely used, by fear of changing its value. There is here a weakness of the Metric System which sooner or later will need to be addressed by metrologists. (Free translation by the Author)

Fig. 4. – The Italian mass standard, the Pt-Ir prototype n. 62, during cleaning-washing at the INRiM.

published only on line [34]. A separate but related problem was raised, that is, whether the national prototypes should also be cleaned and washed before their calibration or not. The Working group on mass standard of the Consultative Committee for Mass and Related Quantities (CCM) was charged with the task of investigating the issue and formulate a recommendation. Eventually, the recommendation suggested to clean and wash also the national prototypes, which the CIPM endorsed [35].

The results of the third verifications were astounding. For an exhaustive account, see [30], from where the relevant diagram was also taken. Overall, all the six official copies of the international prototype showed a drift during the 100 years since their construction amounting to 50 µg (5×10^{-8}) as an average (see fig. 5). Note that the masses of the prototypes were determined during the third verification with a standard measurement uncertainty [36] equal to 2.3 µg. The copies of $\mathfrak{K}$ had been kept in the same vault as the prototype, thus in the same environment conditions, and their usage had been comparable to that of $\mathfrak{K}$. In addition, most of the national prototypes showed a similar, sometimes greater mass increase. The slope of the mass drift due to the accumulation of contaminants amounted to an average of about 1.9 µg year^{-1}([3]). This value was determined by weighing the prototypes prior to cleaning and washing.

A behaviour such as that depicted above casts heavy shades on the ability of an arte-

([3]) Note that the year is neither a SI unit, nor a non-SI unit accepted for usage within the SI. For an interesting discussion highlighting the difficulty inherent in a definition of the year, see [37], p. 20.

fact to define (and realize at the same time) a measurement unit in a reliable way. Indeed, the optimism shown in the twenties by the then Director of the BIPM, the inventor of invar and Nobel laureate Charles Guillaume, according to whom the kilogram definition would remain stable for 10 000 years within 1×10^{-8} (cited in [26]), had been early abandoned. In 1999, the 21st General Conference with its Resolution 7 [38] recommended that the national laboratories continue their efforts towards a definition of the kilogram based on fundamental constants.

Even neglecting contamination, whose effects are almost perfectly reversible by cleaning and washing (but oblige to extrapolate the mass of a prototype in the case of its subsequent usage), the increase of the mass difference between the international prototype and its official copies has no plausible explanation. In addition, the fact that the prototype has by definition a fixed mass of 1 kg implies that formally the responsible for the increase are the official copies (and most of the national prototypes), whereas, at least by statistical evidence, one would be induced to believe that the international prototype is losing mass.

An even worse (and by no means less plausible) scenario is that all the prototypes drift (in different ways) with respect to natural constants. Ironically, many of them are known within some uncertainty, contrary to $\mathfrak{K}$. As we will see, the new SI repairs this situation, establishing a formal framework much closer than the current SI to our perception of the world.

4·2. *Electrical units.* – The ampere came into effect in 1948, when the 9th CGPM [11] adopted resolution 2 of the 41st meeting of the CIPM in 1946 [10], in which the electrical units were defined. The advantages in terms of practicality and rationalization of the electrical units inherent in the adoption of a fourth unit of electrical nature in addition to the mechanical units of the cgs and MKS systems had been demonstrated by Giovanni Giorgi since 1901 [39,40]. Nonetheless, his revolutionary proposal, although supported by eminent scientists internationally recognized, met strong resistances and was not accepted until 1935, when the International Electrotechnical Commission, IEC, abandoned the previous systems in favour of the MKSA [41]. I will not enter here into a detailed discussion of the evolution of electrical units before and after the establishment of the SI. Reference [41], together with its companion [42], constitute a concise yet extremely dense and informative account of the history of the electrical units, including the texts of several key IEC recommendations. Nor I will delve into the intricate relationship between the SI and the still widely used Gaussian units (with several variants). I will just mention that the difficulty comes from the fact that not only units are different among the various systems, but even different quantities and equations are involved. For exhaustive accounts of these aspects, see [43, 44].

What is relevant to the epochal revision of the International System I am considering here is a couple of discoveries which, beside their importance in general physics (they are outstanding examples of macroscopic quantum phenomena) changed the face of metrology. Brian Josephson predicted the effect that took his name in 1962 [45], and Klaus von Klitzing realized the quantum Hall effect, QHE, in 1980 [46]. Both authors were awarded Nobel prizes in physics in 1973 and in 1985, respectively. Key aspects of these discoveries were the identification of two constants underlying the phenomena, the Josephson constant $K_\mathrm{J} = 2e/h$, SI unit $\mathrm{Hz\,V^{-1}}$, and the von Klitzing constant, $R_\mathrm{K} = h/e^2$, unit Ω, respectively (these denominations were introduced subsequently in homage to the discoverers [47]). The universality and independence from the experimental conditions of these constants has now an impressive experimental support [48, 49].

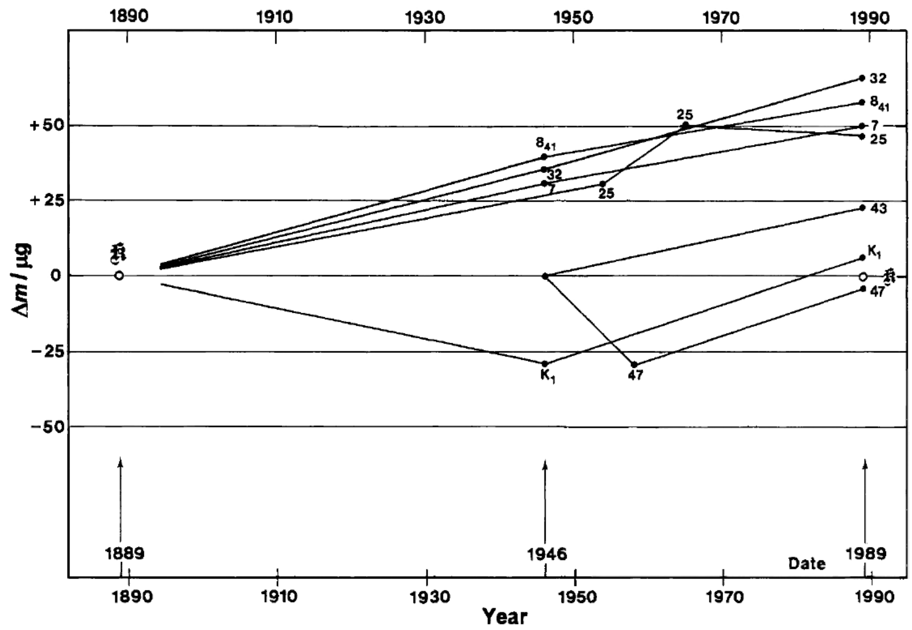

Fig. 5. – Change in mass Δm of the six official copies and of prototype No. 25 with respect to the mass of the international prototype 𝔎 (from [30], courtesy BIPM).

The relevance of these two effects to electrical metrology is that using them it is now possible to realize the volt and the ohm with unprecedented reproducibility and world-wide uniformity, by far exceeding those achievable in terms of the ampere, involving electromechanical experiments. In the eighties of the last century, all the major NMIs had established national voltage standards based on the Josephson junction and were establishing national resistance standards based on the quantum Hall effect. The problem was that, despite the Consultative Committe for Electricity, CCE, had recommended a value for the Josephson constant $K_\mathrm{J} = 483\,594.0\,\mathrm{GHz\,V^{-1}}$ [50], not all NMIs used it, preferring their experimental determinations. In addition, the SI values of both constants were known with an uncertainty greater than their world-wide uniformity, so that the benefits inherent in their usage for the representation of the volt and the ohm were impaired by the uncertainty component associated with their traceability to the SI. Last but not least, their SI values had been changing according to new, better determinations. To remedy this unsatisfactory situation, the CCE appointed two working groups, for the volt and for the ohm, mandated to propose an acceptable solution [51]. These working groups produced two reports, whose essential points were given in a publication [47]. The subsequent CCE meeting in 1988 formulated two recommendations, adopted by the CIPM [52, 53] and sanctioned by the 19th General Conference [54].

The solution proposed and adopted consisted in assigning updated conventional values to the two constants. In addition, these values were to be considered exact, thus neglecting the uncertainty component arising from traceability to the SI. The values were

$$K_\text{J-90} = 483\,597.9\,\mathrm{GHz\,V^{-1}} \quad \text{and} \quad R_\text{K-90} = 25\,812.807\,\Omega,$$

where the subscript 90 serves to denote that the values shall come into effect from January 1st, 1990, and to distinguish these conventional exact values from the physical constants $K_{\rm J}$ and $R_{\rm K}$, which were instead affected at that time by relative standard uncertaintes equal to 4×10^{-7} and 2×10^{-7}, respectively. This implies that perfect representations of the volt and the ohm, realized by means of the Josephson and quantum Hall effect and using $K_{\text{J-90}}$ and $R_{\text{K-90}}$, would have the same associated relative uncertainties. However, comparison and calibrations within the domain of electrical quantities could be carried out neglecting these uncertainties, which resulted in smaller uncertainties. This is exactly what happened until the revision of the SI. For example, the Calibration and Measurement Capabilities in voltage and resistance, as declared by the NMIs in appendix C of the Key Comparison Database, KCDB [55], of the BIPM, at the date of writing are determined according to this criterion (see, *e.g.*, [56]).

It is worth noting that $K_{\text{J-90}}$ is greater by about 8×10^{-6} than the value recommended in 1972 by the CCE, which represents a considerable discontinuity.

It is thus evident that the situation was not satisfactory. In addition, since 1990 the recommended values of h and e, and therefore of $K_{\rm J}$ and $R_{\rm K}$, have further changed, whereas $K_{\text{J-90}}$ and $R_{\text{K-90}}$ are unchanged. As a consequence, $K_{\text{J-90}}$ and $R_{\text{K-90}}$ are now inconsistent with respect to $K_{\rm J}$ and $R_{\rm K}$ by about 1×10^{-7} and 1.8×10^{-8}, respectively, being their respective relative standard uncertainties of 6.1×10^{-9} and 2.3×10^{-10}. This inconsistency impacts of course on the national representations of the volt and the ohm.

4·3. *Physical constants.* – Fundamental physical constants constitute in many respects a special category among physical quantities. Without entering in the debate on which and how many constants are really fundamental, I will consider as authoritative the list given in [57].

The constants are indeed cornerstones in our understanding of nature. From the viewpoint of measurement, each of them constitutes a neat example of ideal measurand, in that we do not doubt that they have a precise and unique value, represented by a real number. Yet, most of them were imperfectly known at the time of the redefinition.

One exception was the magnetic constant or permeability of free space μ_0. This constant was fixed in 1948 by virtue of the definition of the ampere in the International System of Units, SI [14], and has the value

$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{N\,A^{-2}} = 12.566\,370\,614\ldots\times10^{-7}\,\mathrm{N\,A^{-2}}.$$

This value was exact, although, at least in principle, affected by the infinitesimal uncertainty associated with π(4).

There was a second exception, that is the speed of light in vacuum c_0. This was fixed in 1983 by the 17th General Conference on Weights and Measures, CGPM, with

(4) At the date of 11 November 2016, 22 459 157 718 361 (*i.e.*, in a more picturesque description, $\approx\pi^e$ trillion) decimal digits of π were known [58]. But π is an irrational (actually, a transcendental) number, so that its next unknown decimal digit(s) cannot be inferred from those we know. So, our knowledge of π is not complete and never will, although the uncertainty in its value is so low that it can be neglected in all practical applications. However, this pale shade on something believed absolutely exact carries a high conceptual significance. Nobody doubts that π *is* exact, in the sense that it possesses a well-defined, unique value made of an infinite numbers of digits, whichever is the representation chosen, decimal, binary, exadecimal, etc. Yet, this unique value, that we can define exactly, is unknown.

Resolution 1 (see [14] appendix 1), by which the metre was re-defined in terms of the speed of light and the unit of time, the second. The effect was that c_0 was assigned the *exact value*

$$c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}.$$

The reason was that it had been measured to an accuracy exceeding that of the realization of the metre at that time [59]. Exact means that an infinite sequence of zeros has to be imagined after the last significant decimal digit. It is worth noting that, contrary to μ_0, c_0 is known exactly both in practice and in principle.

These two definitions impacted on another constant which is a monomial combination of μ_0 and c_0, *i.e.* the electric constant, or permittivity of free space

$$\epsilon_0 = 1/\mu_0 c_0^2 = 8.854\,187\,817\ldots \times 10^{-7}\,\mathrm{F\,m^{-1}},$$

which became exact as well. As a further consequence, also the characteristic impedance of vacuum Z_0, defined as

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad \text{or} \quad Z_0 = \mu_0 c_0,$$

became exactly defined.

There were some more constants exactly defined. I already discussed $K_{\text{J-90}}$ and $R_{\text{K-90}}$ as well as c_0, μ_0, ϵ_0 and Z_0. The list is complete by adding some quantities relevant to the amount of substance, namely:

- the molar mass of ^{12}C, $M(^{12}\mathrm{C})$, which is equal to $12 \times 10^{-3}\,\mathrm{kg\,mol^{-1}}$,
- the related molar mass constant $M_\mathrm{u} = M(^{12}\mathrm{C})/12 = 10^{-3}\,\mathrm{kg\,mol^{-1}}$,
- the relative atomic mass of ^{12}C, $A_\mathrm{r}(^{12}\mathrm{C}) = 12$.

All the other constants were to be determined experimentally. The result was that uncertain values, when expressed in terms of exactly defined SI units, were available for universal constants of nature. Yet, these units were known to be far from stable or universal.

4·4. *The kelvin.* – The main concern about the kelvin was the evidence, gained from the results of an international comparison of water triple point cells [60], that the temperature of the triple point T_tpw depends of the isotopic composition of water. This led the CIPM to adopt from the CCT a recommendation which became CIPM Recommendation 2 (CI-2005) "Clarification of the definition of the kelvin, unit of thermodynamic temperature" [61]. In the recommendation the definition of the kelvin is specified in terms of a given isotopic composition of water, *i.e.*, that of the reference material Vienna Standard Mean Ocean Water, VSMOW, of the International Atomic Energy Agency, IAEA. Several efforts were devoted to the task of accurately determining the isotopic corrections (see, *e.g.*, [62-64]), with the result that the reproducibility of the temperature of the triple point of water is better than 50 μK. This value is well below any foreseeable need in thermometry, so that the situation of this discipline could hardly be considered unsatisfactory. As we will see, the redefinition of the kelvin in the revised SI led to a greater uncertainty for T_tpw.

4˙5. *Dimensionless quantities.* – Seven dimensions are conventionally defined, corresponding to the SI base quantities. They are time, length, mass, electric current, thermodynamic temperature, amount of substance and luminous intensity, symbols **T**, **L**, **M**, **I**, **Θ**, **N** and **J**, respectively. The dimension of any quantity is expressed by a dimensional equation

$$\dim Q = \mathbf{T}^{\alpha}\mathbf{L}^{\beta}\mathbf{M}^{\gamma}\mathbf{I}^{\delta}\boldsymbol{\Theta}^{\epsilon}\mathbf{N}^{\zeta}\mathbf{J}^{\eta},$$

where α, β, etc. are negative or positive, appropriate, usually small, integers.

Dimensional analysis has unquestionable merits in science and engineering. The Buckingham π theorem [65] is at the basis of important applications, for example many dimensionless parameters widely used in fluid dynamics. However, the doubt remains that dimensions are simply a duplication of base quantities (or units), and the term is lacking a convincing definition, even in the International vocabulary of metrology (see [3], definition 1.7), which has led to questioning the usefulness of the very concept [66]. As a matter of fact, when checking the dimensional correctness of an expression, symbols for units (or for quantities) serve equally well. In addition, confusion is easily engendered among the symbol for units, quantities and dimensions, although the fonts are different.

When for a quantity all the dimensional exponents are zero, the quantity is said to be dimensionless or of dimension one. This occurs in a number of instances, for example when the quantity is the ratio of two quantities of the same kind, such as amount fraction in chemistry, or when the quantity is a number of items such as molecules, or states. Angles, plane and solid, are considered to be the ratio of two lengths and two squared lengths, respectively, and are as such dimensionless in the SI. Nonetheless, their units, radian, symbol rad, and steradian, symbol sr, respectively, occupied since the establishment of the system in 1960 such a special role that a third category of units besides base and derived was created for them. Indeed, they both were considered as supplementary units, that is, neither base nor derived. However, the situation was far from being satisfactory. On the one hand, the nature of what really is a supplementary unit was never clarified, which casts some shadow on the consistency of the system. On the other hand, considering radian and steradian as base units posed a number of conceptual and formal difficulties, and simply defining them as derived units seemed too reductive with respect to their role in the SI. Eventually, in 1995 the 20th General Conference decided to endorse a 1980 CIPM recommendation (Recommendation 1 (CI-1980) [67], p. 24) and to consider radian and steradian as dimensionless derived units "the names and symbols of which may, but need not, be used in expressions for other SI derived units, as is convenient" [68], thus eliminating supplementary units from the SI.

Dimensionless quantities, and especially angles, are dangerous in that they can induce errors in various ways. To give just an example, relative uncertainties associated with estimates of dimensionless quantities are ambiguous and can easily be misinterpreted. As concerns angles, the fact that the corresponding units "may but need not" be included among the units of quantities involving them implies that, for example, action and angular momentum share the same SI unit joule second, J s, and have the same dimension. Similarly, energy and torque are both expressed in joule (although for torque the explicit expression N m is usually preferred). Another striking example concerns the base unit of luminous intensity, the candela, and that of luminous flux, the lumen, symbol lm, related by lm = cd sr or, in SI formally correct writing, lm = cd([5]). The fact that different quantities have the same units is not *per se* a problem, as we all learned that the unit alone does not characterize enough the corresponding quantity. Nonetheless, I feel that a good system of units should be such that occurrences of this type are minimized.

Further problems arise with periodic phenomena, where the angular velocity ω has SI unit radian per second, rad/s, and the frequency has the SI unit hertz, Hz, "implying the unit cycles per second" [14].

In sect. **10** I will discuss an attempt to give a more rational basis to dimensionless quantities in the SI, attempt that so far has been unsuccessful.

5. – Early proposals

From the discussion in the previous sect. **4** it clearly emerges that the main problem was by far that of the kilogram. As mentioned in sect. **4**˙1.1, the circles of international metrology were well aware of the inadequacy of a definition (and realization) of a unit based on an artefact. The most intuitive idea for a better definition was of course that of using (indeed, to come back to) a natural standard, possibly better than the litre of distilled water at its melting temperature (or at its maximum density) of the origins. In the sixties, the possibility of a natural standard for the mass unit was seen as a utopia. It is not a case that one of the first suggestions in this direction, incidentally by an Italian scientist, Claudio Egidi, at that time at the Istituto Elettrotecnico Nazionale "Galileo Ferraris" IENGF, now INRiM, was entitled "Phantasies on a Natural Unity [sic] of Mass" [71], corrected in [72]. Egidi's proposal, or dream, suggested a natural standard of mass having cubic shape, made of a monocrystal of some monoisotopic material, or of a non-monoisotopic material, such as germanium, which would need to be artificially enriched so as to obtain an essentially monoisotopic material. The faces of the cube would be aligned with the crystalline planes. The artefact would be compared with the international prototype $\mathfrak{K}$ so as to have essentially the same mass, *i.e.*, 1 kg. The number of atoms would be determined either by counting the atoms aligned along the crystallographic axes or, alternatively, by measuring the lattice parameter of the crystal, *i.e.*, the distance between two planes of the crystal lattice. The paper goes on suggesting:

> From this moment on, we can say: "The kilogram is the mass of K atoms of ...".

As a further benefit of this suggested definition, a very accurate estimate of the Avogadro constant ("number" in the paper) would be obtained. The estimate of the number K of atoms could subsequently be artificially assigned an uncertainty equal to zero, thus taking that value as the definition of the kilogram. The procedure was claimed to be analogous to that followed for the new (for that time) definition of the metre.

The transition from dreams to reality was enabled by the improvement on the knowledge of fundamental constants, and will be addressed in sect. **8**.

6. – The proposal of 2005

In February 2005, a seminal paper threw a rock in the somehow somnolent pond of metrology [73]. The title itself, *Redefinition of the kilogram: a decision whose time has*

([5]) Incidentally, a debate surfaces from time to time about the convenience of changing the base unit from candela to lumen, so far with no success [69, 70].

come, was shaking by itself. The paper was authored by some of the most prominent protagonists of international metrology. Ian Mills, at that time, was the President of the Consultative Committee for Units. Terry Quinn had been for many years (1989 to 2004) the Director of the BIPM, and still was active in many important committees as well as in experimental determination of fundamental constants. Peter Mohr and Barry Taylor (NIST) were in the CODATA Task Group on Fundamental Physical Constants [74]. Last but not least, Edwin Williams was the head of the NIST experiment of the Kibble balance (at that time, still the watt balance) by which the most accurate determination of the Planck constant h had been made available.

The content of the paper was unequivocal: it was the belief of the authors that the conditions were mature for the long-sought redefinition of the kilogram, and that this could be sanctioned by the next General Conference, in 2007. They argued that it could be possible to fix the Planck constant h or the Avogadro constant N_A and relate the chosen constant to the mass unit through a suitable experiment, similarly to what had been made in 1983 for the speed of light in vacuum c_0 and the metre [75]. The choice between h and N_A is mostly a matter of convenience, as the two constants are related through the expression

$$N_\mathrm{A} = \frac{c_0}{2}\frac{A_r(\mathrm{e})\alpha^2}{R_\infty}\frac{M_\mathrm{u}}{h}\,, \tag{12}$$

where $c_0 = 299\,792\,458\,\mathrm{m\,s^{-1}}$ is the (exact) speed of light in vacuum, $A_r(\mathrm{e}) = 12m_\mathrm{e}/m(^{12}\mathrm{C})$ is the relative atomic mass of the electron, α is the fine-structure constant and R_∞ is the Rydberg constant [73]. M_u is the molar mass constant, exactly equal to $1\times10^{-3}\,\mathrm{kg\,mol^{-1}}$.

The uncertainties with which $A_r(\mathrm{e})$, α and R_∞ are known were negligible with respect to those about h and N_A. Therefore, the CODATA values of the two constants of interest were strongly anticorrelated ($r = -0.9993$) so that either could be a suitable candidate for a redefinition of the kilogram. As a further consideration, any experiment that determines one of the two provides the other through expression (12). This expression had been adopted in the 2002 adjustment of fundamental physical constants by CODATA [49] to obtain the recommended value of N_A from the adjusted values of the other quantities involved.

The authors in an appendix to the paper discussed also practical issues, such as possible formulations for the new definition of the kilogram, and the merits of each choice concerning the constant to be adopted as the basis for a redefinition. To summarize here the most important points, choosing the Planck constant —and fixing also the elementary charge e— has the merit of making exact the values of the Josephson and von Klitzing constants, $K_\mathrm{J} = 2e/h$ and $R_\mathrm{K} = h/e^2$, respectively, at that time at the basis of the practical representation of the volt and the ohm. As to the definition, a subsequent paper [76] is much more detailed.

The claim that the conditions for a change of definition were propitious was a rather bold one. Actually, it had been previously recommended (ironically, even by one of the authors) [77] that a suitable relative uncertainty u_r associated with a kilogram derived from a fundamental constant to replace the international prototype should be $u_\mathrm{r} \approx 1\times10^{-8}$. Both of the two candidate constants h and N_A had rather at that time a relative uncertainty of about 1×10^{-7}. But the worst was that the two main experiments involved in the determination of the constants yielded values discrepant by about 1×10^{-6}. The CODATA Task Group had included the data from XRCD experiment in the (weighted) least-squares adjustment by increasing the uncertainty declared by a factor of 2.325 [49]. Nonetheless, there would be great advantages in fixing either of the constants in terms of the decrease of the uncertainties of many other fundamental constants, essentially all those depending on $m(\mathfrak{K})$. For example, by fixing the value of h the relative uncertainty of N_A would become $u_\mathrm{r}(N_\mathrm{A}) = 0.67 \times 10^{-8}$. The electron mass $u_\mathrm{r}(m_\mathrm{e})$ would pass from 17×10^{-8} to 0.67×10^{-8} and that of the elementary charge, $u_\mathrm{r}(e)$, from 8.5×10^{-8} to 0.17×10^{-8}. By fixing N_A the situation would of course be very similar. For a more comprehensive list, see Table 1 in [73].

An important aspect to consider is that the values of the constants would not be affected by the proposal, which would just determine a rearrangement of the uncertainties.

The price to be paid was the loss of the unique virginity of $\mathfrak{K}$, whose mass $m(\mathfrak{K})$ would become subjected to adjustment, and take an initial relative uncertainty $u_\mathrm{r}(\mathfrak{K}) = 1.7 \times 10^{-7}$. This was exactly the relative uncertainty affecting the CODATA 2002 recommended values of both N_A and h. In terms of absolute uncertainty, this amounted to 170 µg, to be compared and, above all, combined with the typical uncertainty in the calibration of a prototype kilogram, somewhere between 2 µg and 3 µg.

The reasoning of the authors was that the advantages in the field of the fundamental constants greatly outweighed the loss in mass metrology. This loss could be minimized by adopting a conventional value for the mass $m(\mathfrak{K})$ of the international prototype, similarly to what had been done since 1990 for the electrical units. This conventional value would be denoted $m(\mathfrak{K})_{07}$ (the authors hoped that the redefinition could take place in 2007) and would serve as a reference for the worldwide mass metrology. The mass community would enter in this way a new situation, the same in which electrical quantities had been for several years and from which the proposal intended to free them. The conventional value $m(\mathfrak{K})_{07}$ would be updated periodically according to the improvement in the determination of h (or N_A).

The reaction of the community of mass metrologists was not favourable. The CCM met in April 2005, two months after the publication of the paper [78]. As most Consultative Committees, the CCM is organised in working groups, the one relevant to the issue of the redefinition being the WG "mass standards" ([6]). The WGs meet typically prior to the plenary session of the CC. The proposed redefinition was presented by one of the authors, TJQ, and discussed by the delegates, at both the WG and the plenary meetings. Concerns were expressed by most of the delegates, and written positions were also presented by some NMIs and by EURAMET, the European metrology organization. Doubts were raised on technical issues, but especially on the comparatively poor knowledge about the constants intended to serve as defining quantities.

As to the technical reasons, it must be remarked that weighings (perhaps the most widespread kind of measurement also nowadays) have been traditionally carried out in air, as historically there has been no special need for in-vacuum weighings, except perhaps for some sophisticated scientific applications. After all, the prototype itself was kept in atmosphere, and the vast majority of weighings are intended for trade. As a consequence, at the time of the proposal most NMIs had no balances capable of operating in vacuum. Commercial, top-class mass comparators capable of consistent and reasonably easy operation in vacuum are recent. To give an example, IMGC had a vacuum-operating 1 kg mass comparator since 1997, obtained from a commercial comparator placed inside a

([6]) An *Ad hoc* Working Group on changes to the SI, CCM WGSI, was subsequently established in 2006, as in all the CCs concerned with the changes.

home-designed airtight enclosure. However, the mass comparator which is currently in use in most NMIs became operative at the INRiM only in 2011.

The new interest towards weighings in vacuum was motivated by the fact that the Kibble balance, the device developed to determine the Planck constant, operates in vacuum, so that the problem arose of the transfer from vacuum to air and vice versa of the standards. The operation poses subtle problems connected with sorption and desorption from the surface of the standards, involving reversible and non-reversible mass changes depending in turn on the material and its surface [79]. The phenomena behind the mass changes in the transfer were far from being well understood and quantified at the time of the proposal of redefinition of the kilogram.

To add to the existing difficulties, an issue had arisen concerning the air buoyancy. This effect implies a correction depending on the volume difference between the standards under comparison and the air density. The latter has been traditionally determined indirectly, by measuring the physical (temperature and pressure) and chemical (concentrations of the various components) parameters of air. These data are used in a formula for the density of moist air agreed upon at the international level in 1981 [80] and updated in 1991 [81], essentially to take into account a new and better estimate of the molar gas constant [49], and the change in the definition of the international temperature scale from IPTS68 to ITS90. That version was in use at the time of the proposal of redefinition of the kilogram. In practice, only few of the chemical parameters are measured (H_2O and CO_2 concentration), the others being either inferred (oxygen and carbon dioxide are considered to sum to a constant value) or assumed to be constant, with values taken from the literature. More or less at that time, a different, direct way of measuring the air density was reaching maturity. This technique, pioneered in the eighties [82], was based on two artefacts having mass and surface area values as close as possible and a volume difference as large as possible. Knowing the masses and the volumes of the two artefacts it is possible to determine the air density from the apparent mass difference indicated by the balance with an uncertainty smaller than that of the so-called thermodynamic formula. The two methods for measuring the air density were shown to be inconsistent, and the suspicion fell on the molar fraction of argon [83], recently redetermined [84].

When the proposed redefinition of the kilogram was published, all the above was still matter of debate and investigation, and only in 2008 the formula for air density was updated [85]. Nowadays the direct determination of air density is commonly carried out in most NMIs by weighing the air-density artefacts during the mass comparisons by using balances designed to host on a carousel and compare in vacuum or in ambient air up to eight bodies (fig. 6).

To conclude with the perplexities of technical nature, the general feeling during the 2005 CCM was that the realization and the transfer of the new definition of the mass unit would present technical difficulties suggesting to postpone its approval.

The objections of conceptual nature were by far more compelling. The main concern was that the most recent and accurate determinations of h (as well as of N_A), *i.e.*, the Kibble-balance experiment (NIST 1998, [86]) and the XRCD (WGAC 2003) were inconsistent [49]. The situation concerning the two experiments was

$$h_{\mathrm{WGAC}} = 6.6260762(21) \times 10^{-34}\,\mathrm{J\,s} \quad \text{and} \quad h_{\mathrm{NIST}} = 6.62606891(58) \times 10^{-34}\,\mathrm{J\,s}.$$

Relative uncertainties were thus 3.2×10^{-7} and 8.8×10^{-8}, respectively, to be compared with the inconsistency, amounting to 1.1×10^{-6}. Yet, the CODATA recommended value was $h_{\mathrm{CODATA}} = 6.626\,069\,3(11) \times 10^{-34}\,\mathrm{J\,s}$, implying a relative uncertainty equal to

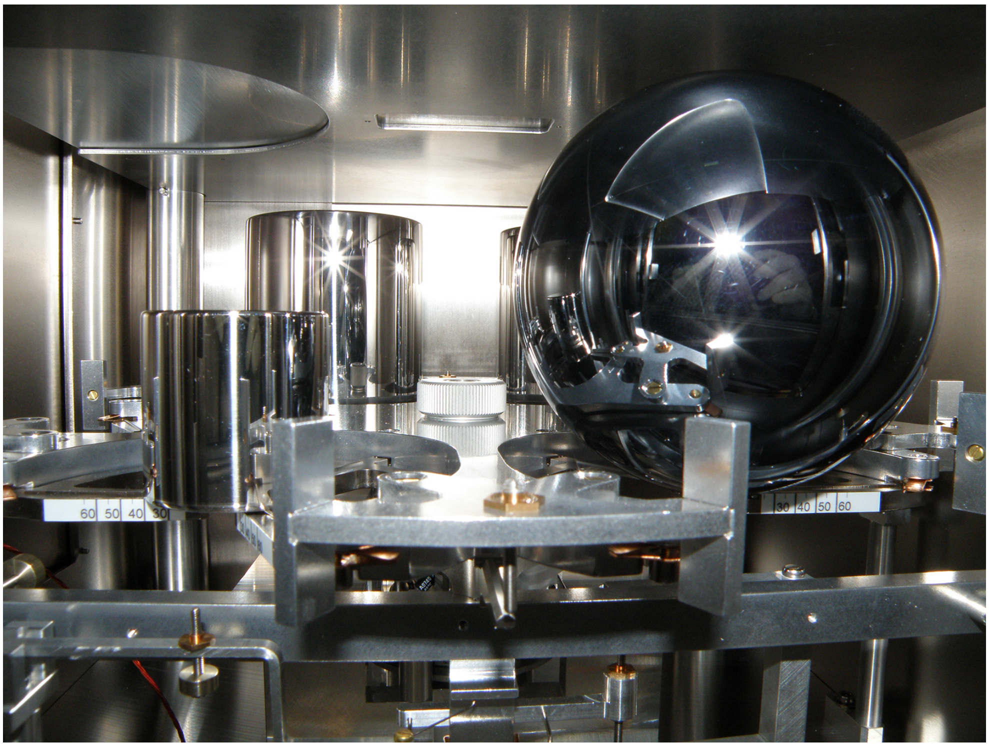

Fig. 6. – A silicon sphere, a Pt-Ir kilogram and the artefacts for the direct determination of the air density on the carousel of a modern mass comparator.

1.7×10^{-7}. Now, this latter uncertainty seemed too small for a value pretended to be representative of the true state of knowledge about the Planck constant and some doubts were raised during the 9th meeting of the CCM in 2005 about the way CODATA processes the input data. Typically, in the case of inconsistent input data for a given constant, the CODATA strategy was and still is to inflate the input uncertainties as declared by the experimenters by a multiplicative factor so that the so-called Birge criterion is satisfied. The Birge criterion is $\chi^2_{\text{obs}} \leq \nu + \sqrt{2\nu}$, where χ^2_{obs} is the χ^2 calculated from the data set and ν is the degrees of freedom. The proponents of the redefinition further argued that the 1×10^{-8} target uncertainty considered in 1991 for a redefinition of the kilogram was perhaps too strong a constraint and that by relaxing it to, say, 5×10^{-8}, the distance between the uncertainty of the CODATA recommended value and the relaxed constraint would not be too great.

In addition, the widespread suspicion was that the main reason for the inconsistency was the determination of the molar mass $M(\text{Si})$ of natural silicon, a mixture of three stable isotopes, ^{28}Si, ^{29}Si and ^{30}Si. The uncertainty in the molar mass $M(\text{Si})$ was the dominant contribution to the uncertainty of N_{A}, due essentially to the determination of the isotopic composition of Si, on which suspicions were general. The issue was eventually solved by building spheres made essentially of enriched ^{28}Si. On the side of the Kibble balance, in a paper published in November that same year 2005, NIST confirmed their 1998 value for h with a smaller uncertainty [87].

Eventually, the CCM was not persuaded that the moment be the best one for a redefinition, thinking rather the contrary. The reasons of those in favour of an immediate redefinition were good and understandable, their concern being that the difficulties inherent in the determination of the constants to a higher accuracy could delay indefinitely the redefinition of the kilogram. The reasons of the opponents were mixed. Some delegates (the writer among them) were wholeheartedly in favour of a redefinition as soon as possible, considering that the additional uncertainty on the international prototype would not harm the good consistency of mass measurements at the international level but thought that the inconsistency between the two leading experiments should first be better understood and resolved.

In connection with the mentioned inconsistency, the proposal of a conventional value for $m(\mathfrak{K})$ was strongly opposed. The risk inherent in an immediate redefinition associated with a conventional value was a discontinuity in the value of the unit should further experiments on h (or N_A) change significantly their recommended values. Now, a discontinuity in the value of a unit (or in a scale) is not *per se* a drama (provided that it is sufficiently small for practical applications), and actually there were precedents. For example, in the transition from the International Practical Temperature Scale of 1968, IPTS-68, to the International Temperature Scale of 1990, ITS-90, the temperature of the boiling point of water decreased by some 26 mK [88]. However, it seemed appropriate to wait and see. Other delegates were reluctant simply by inertia and diffidence against novelty. Among the latter category, some NMIs had strategic reasons, as they wanted to gain more visibility in the merit of the redefinition and needed some more time to progress in their determination of the Avogadro constant.

The various reasons within the CCM were encapsulated in a recommendation G1 (2005) to the CIPM: "Conditions for a new definition of the kilogram", whose essential points were:

> *The Consultative Committee for Mass and Related Quantities (CCM),*
>
> ...
>
> *recommends*
>
> – that the following conditions be met before the kilogram is redefined with respect to a fundamental constant:
>   1) there are no significant unresolved discrepancies between results from independent experiments,
>   2) the relative standard uncertainty of the best realization of the definition of the kilogram does not exceed two parts in 10^8, at the level of one kilogram,
>   3) the results of a sufficient number of independent experiments are available with the required uncertainty,
>
> – that the CODATA recommended value be adopted for the relevant fundamental constant,
>
> – ...
>
> – that a *mise en pratique* for the realization of the new definition of the kilogram be drawn up that includes recommendations concerning the various linking experiments, as well as recommendation for the continuing use of the present artefact to maintain the present excellent world-wide uniformity of mass standards.

A *mise en pratique* is a document illustrating the recommended way (or the recommended ways) to realize the definition of a unit. The *mise en pratique* was a novelty for the kilogram, as the realization of the definition coincided with the definition itself. It existed for the metre since 1983, when this unit was defined in terms of the speed of light in vacuum c_0.

The change of definition of the kilogram has of course repercussions on other base units, the ampere, the candela and the mole (see fig. 1). That same year 2005 the corresponding CCs met, and the CCEM and CCQM discussed the impact of the proposal on their communities. They came out with recommendations to the CIPM, more or less in the same format, both generically encouraging further research and suggesting to defer a decision to the 2011 24th CGPM. The CCEM suggested to consider at that time to fix both the Planck constant and the elementary charge [89], and the CCQM to fix both the Planck and the Avogadro constants [90]. The CCPR did not react to the proposal, possibly because of the minor impact of the proposal itself on the candela. The Consultative Committee for Thermometry, CCT, discussed a proposal concerning the redefinition of the kelvin in terms of the Boltzmann constant k [91].

Also the CODATA Task Group on Fundamental Constants, TGFC, met at the BIPM and issued a Recommendation supporting the simultaneous redefinitions of the kilogram, the ampere and the kelvin in terms of h, e and k, respectively, and suggesting to give consideration to the redefinition of the mole in terms of N_A [92].

The meeting of the Consultative Committee for Units that year was largely devoted to the proposed redefinition of the kilogram and possibly of the ampere, the kelvin and the mole [93]. Reports and recommendations from the relevant Consultative Committees and the CODATA TGFC were included as working documents, and the debate anticipated most of the topics that would be developed in the subsequent years in the literature and that will be described later on in this paper. Eventually the Committee approved the Recommendation U 1 (2005) "On possible changes to the definitions of the kilogram, the ampere, the kelvin and the mole"

> *The Consultative Committee for Units (CCU),*
> *considering*
>
> - ...
> - the impact on metrology of the application of the Josephson and quantum-Hall effects;
> - the consensus that now exists on the desirability of finding ways of defining all of the base units of the SI in terms of fundamental physical constants so that they are universal, permanent and invariant in time;
> - ...
> - the broad view that has emerged from discussions at these meetings of Consultative Committees and the CODATA Task Group, that if changes do take place in the definitions of the kilogram, the ampere and the kelvin, they should all take place at the same time;
> - ...
>
> *requests* that
>
> - the CIPM approve in principle the preparation of new definitions and *mise en pratiques* [sic] of the kilogram, the ampere and the kelvin so that if the results of experimental measurements are indeed acceptable, all

having been agreed with the various Consultative Committees and other relevant bodies, the CIPM can prepare proposals to be put to Member Governments of the Metre Convention in time for possible adoption by the 24th CGPM in 2011;

– the CIPM give consideration to the possibility of redefining, at the same time, the mole in terms of a fixed value of the Avogadro constant;
– the CIPM prepare a Draft Resolution that may be put to the 23rd CGPM in 2007 to alert Member States to these activities;
– ...

The essence of the CCU recommendation was thus to keep impetus to the proposal, while recognising that the time had not yet come for the revision.

The CIPM met that year in October [61] and issued the Recommendation 1 (CI-2005), significantly entitled "Preparative steps towards new definitions of the kilogram, the ampere, the kelvin and the mole in terms of fundamental constants". The recommendation was obviously based on those prepared by the Consultative Committees. In it, the CIPM "approves, in principle, the preparation of the new definitions, as requested by the CCU in its Recommendation cited above".

The way was paved for the revision of the International System, when the conditions would become appropriate.

7. – The 2006 comprehensive proposal

All the debates and discussion of the previous year in the various Consultative Committees, the CODATA TGFC and the CIPM were in a sense formalized in a paper by the same authors of the 2005 proposal in which that proposal is extended to include redefinition of the kilogram, the ampere, the kelvin and the mole in terms of the Planck constant h, the elementary charge e, the Boltzmann constant k and the Avogadro constant N_A, respectively [76].

The extended proposal takes a firm position concerning the alternative between the Planck and the Avogadro constants as the funding constant for the definition of the kilogram, in favour of h. Three are the reasons to prefer h rather than N_A. First, as already mentioned, fixing h and e has the advantage that the Josephson constant $K_\mathrm{J} = 2e/h$ and the von Klitzing constant $R_\mathrm{K} = h/e^2$ take exact values, with great advantages for the metrology of electrical quantities. Second, using h to define the kilogram gives the opportunity of using N_A for the definition of the mole. Last but not least, from the viewpoint of fundamental physics it is desirable that, similarly to the speed of light in vacuum c_0, also h be exactly known, as this constant plays in quantum mechanics a role similar to that played by c_0 in relativity.

The price to pay is that, beside the kilogram, also the other defining invariants of the (then) current SI, *i.e.*, the molar mass of carbon 12 $M(^{12}\mathrm{C})$, μ_0 (and the related ϵ_0 and Z_0) and the temperature of the triple point of water T_tpw would take an uncertainty.

7·1. *Proposed definitions.* – The authors proposed two possible kinds of definitions, called explicit-unit and explicit-constant definitions.

7·1.1. Explicit-unit definition. In the explicit-unit definition, the unit is explicitly defined in terms of a quantity of the same kind which is a monomial expression containing, among auxiliary units, the defining constant, whose value is in turn implicitly defined itself. Adopting the notation given in expression (1),

$$[Q_{\mathrm{B}}] = \prod [Q_i]^{p_i} \left(\frac{C}{\{C\}} \right)^e , \tag{13}$$

where $[Q_{\mathrm{B}}]$ is the base unit, $[Q_i]$ are auxiliary units and C is the constant, normalized to the value one by division by its numerical value $\{C\}$ [94, 95].

The SI value of the defining constant is thus implicitly defined, which can be made explicit by solving expression (13) for C

$$C = \{C\} \left(\frac{[Q_{\mathrm{B}}]}{\prod [Q_i]^{p_i}} \right)^{1/e} . \tag{14}$$

Expression (13) encompasses the case in which the defining constant is a quantity different from the defined unit, which is the case with all the SI base units except the second.

This kind of definition was introduced in the SI with definition (8) of the ampere, followed by definition (6) of the metre. If the concept of constant is broadened to include the mass of the prototipe kilogram $m(\mathfrak{K})$, the hyperfine splitting frequency of the cæsium ground state $\Delta\nu_{\mathrm{Cs}}$, the triple point of water T_{tpw}, the spectral luminous efficacy K_{cd} and the molar mass of $^{12}\mathrm{C}$ $M(^{12}\mathrm{C})$ this kind of definition can be considered typical of the SI since its origin. As a matter of fact, all the definitions from (2) to (9) follow this broad scheme, although their wordings are considerably different and reflect the different epochs at which they were introduced.

To the same scheme belong also the explicit-unit definitions proposed in the paper under consideration. For example, for the ampere it is suggested that

> The ampere is the electric current in the direction of the flow of exactly $1/(1.602\,176\,53 \times 10^{-19})$ elementary charges per second.

This is straightforwardly

$$\mathrm{A} = \frac{e}{\{e\}}\,\mathrm{s}^{-1},$$

thus in the format of expression (13), from which the value of the defining constant, the elementary charge e, is defined, as per expression (14), as

$$e = \{e\}\,\mathrm{A\,s} = 1.60217653 \times 10^{-19}\,\mathrm{C}.$$

In the expressions above, as well as in the remainder of this section, the numerical value $\{e\} = 1.602\,176\,53 \times 10^{-19}$ is the one recommended in the CODATA 2002 adjustment for the elementary charge [49], and is of course now obsolete. The same applies to the values of the other constants considered.

If the definition of the ampere is straightforward and easily understood, things become more complicated with the kilogram. The first suggested definition is:

> The kilogram is the mass of a body whose equivalent energy is equal to that of a number of photons whose frequencies sum to exactly $[(299\,792\,458)^2/66\,260\,693] \times 10^{41}$ Hz.

The definition is far from being transparent and widely understandable. The suggested equivalent alternative does not improve the accessibility:

> The kilogram is the mass of a body whose de Broglie-Compton frequency is equal to exactly $[(299\,792\,458)^2/(66\,260\,693 \times 10^{-34})]$ Hz.

In both cases, the underlying physical laws are $E = mc_0^2$ and $E = h\nu$, from which $\nu = mc_0^2/h$ and

$$\mathrm{kg} = \frac{h}{\{h\}}\left(\frac{\{c_0\}}{c_0}\right)^2 \mathrm{s}^{-1}, \quad h = \{h\}\frac{c_0^2}{\{c_0\}^2}\,\mathrm{kg\,s} = 6.626\,069\,3 \times 10^{-34}\,\mathrm{J\,s}.$$

The de Broglie-Compton frequency of a body having mass m is $\nu_m = c_0/\lambda_m$ where $\lambda_m = h/(mc_0)$ is the Compton wavelength. The first alternative is an attempt to avoid the ridiculously small and unphysical value of the Compton wavelength for a body of one kilogram, about seven orders of magnitude smaller than the Planck length.

7·1.2. Explicit-constant definition. The alternative kind of definitions, the explicit-constant definition, implicitly defines the unit by explicitly giving the exact value of the defining constant. In this scheme, significantly departing from the SI-traditional explicit-unit definition scheme, all base units have the same template:

> The [name of base unit], unit of the [name of base quantity], is such that the [name of fundamental constant] is exactly [value of fundamental constant].

For the kilogram the definition would be: the *kilogram*, unit of *mass*, is such that the *Planck constant* is exactly $6.626\,069\,3 \times 10^{-34}$ J s.

For the other base units similar definitions hold, each establishing a one-to-one correspondence between the unit and the defining constant. It is straightforward to show that also with this kind of definition both the unit can be realized in terms of the defining constant and the constant is fixed by the definition. For the kilogram and the Planck constant

$$h = 6.626\,069\,3 \times 10^{-34}\,\mathrm{J\,s} = \{h\}\,\mathrm{m}^2\,\mathrm{kg\,s}^{-1}$$

and

$$\mathrm{kg} = \frac{h}{\{h\}}\,\mathrm{m}^{-2}\,\mathrm{s}.$$

Therefore, using any experiment able to relate the mass of a body to h (currently the Kibble balance and the XRCD experiment) the kilogram can be realized. Since the second s and the metre m can also be realized in the laboratory, by fixing the value of h the definition completely specifies the kilogram and suggests means to realize it in practice. A marginal but interesting advantage of this kind of definition is that the same format can be used for all the base units, thus greatly improving the formal consistency of the system. This positive feature suggests to modify accordingly also the definitions of the units not subjected to redefinition.

7.1.3. Explicit-constant generalized. Both kinds of definitions discussed in sects. **7**.1.1 and **7**.1.2 link a base unit to a specific defining constant. This feature looks like an unnecessary constraint and can be removed. In expression (13) each of the quantities in $\prod[Q_i]^{p_i}$ can be expressed in terms of the relevant defining constant, so that eventually expression (13) becomes

$$[Q] = \prod [C_i]^{e_i}\,, \tag{15}$$

where the subscript B for the unit has been dropped since the expression applies as well to base and derived quantities.

The authors proposed this third alternative, in the following form:

> the International System of Units, the SI, is the system of units scaled so that the
>
> 1) ground state hyperfine splitting transition frequency of the cæsium 133 atom $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ is 9 192 631 770 Hz,
> 2) speed of light in vacuum c_0 is 299 792 458 metres per second,
> 3) ...

and so on for all the seven constants. All the individual definitions of the base units would be abrogated in this scheme, as no longer necessary, being the assignment of exact values to the seven constants sufficient to define the architecture of the system. The sufficiency is demonstrated by the previous discussions in which the seven base units of the current SI are specified in terms of the seven constants.

This audacious proposal opens at least two interesting perspectives. First, there is no need to further distinguish between base and derived units.

Second, the aspect of how many and which constants to choose gains momentum. Actually, in the same paper a mention is made to the possibility of using a different set of constants, such as the Josephson constant $K_{\mathrm{J}} = 2e/h$ and the von Klitzing constant $R_{\mathrm{K}} = h/e^2$ instead of the Planck constant h and the elementary charge e. Also the Boltzmann constant k could be replaced by the Stefan-Boltzmann constant $\sigma = (2/15)\pi^5 k^4/(h^3 c_0^2)$ and the Faraday constant $N_{\mathrm{A}}e$ could be used to replace the Avogadro constant N_{A}. Consideration is also given to the possibility that in a future the Rydberg constant R_∞ replaces the cæsium ground state transition frequency $\Delta\nu(^{133}\mathrm{Cs})_{\mathrm{hfs}}$ which, as already discussed, is definitely not a universal constant, but rather an invariant natural standard, like the triple point of water T_{tpw}.

8. – Conditions that made the 2006 proposal possible

In this section I will only sketch the experiments by which the relevant constants were measured. Detailed accounts are beyond the scope of this paper and can be found in the enormous literature on this topic. Several relevant papers are also cited here.

The idea of using the fundamental constants of physics as a sound basis for a system of units dates back to Maxwell [96]. The first realization of the old dream was in 1983, when the speed of light in vacuum c_0 was adopted to define the metre. The problem of the kilogram was more difficult, as demonstrated by the time span of 35 years between the redefinition of the metre and that of the kilogram.

In the meantime, several proposal [97-101] considered the general possibility of a system of units based on universal constants.

On the experimental side, the proposal of Egidi's paper [71] mentioned in sect. **5** was at the basis of the very idea underpinning the X-Ray-Crystal-Density (XRCD) experiment, one of the two (the other being the Kibble balance) from which the most accurate values of the Planck and Avogadro constants were obtained and, as such, one of the two which paved the way to the redefinition of the kilogram (and the mole).

In XRCD, the sample is a near-perfect sphere of monocrystalline, isotopically enriched silicon (99.999% ^{28}Si [102]). The number of silicon atoms in this sample is $N(\mathrm{Si}) = 8V/a_0^3$, V and a_0^3 being, respectively, the volumes of the sphere and of the unit atomic cell, containing 8 atoms. The amount of silicon is $n(\mathrm{Si}) = m/M$, where m and M are the mass of the sphere and the molar mass of silicon. Therefore the definition of the Avogadro constant may be written as

$$N_{\mathrm{A}} = \frac{N(\mathrm{Si})}{n(\mathrm{Si})} = \frac{8V}{a_0^3}\frac{M}{m}\,,$$

which is the basic model describing the measurement principle for the XRCD experiment. With this experiment, the Avogadro constant is determined in terms of the kilogram, the second and the metre. Therefore the kilogram can be determined in terms of the second, the metre and the Avogadro constant.

The pioneering NIST (NBS at that time) work [103, 104] provided relative uncertainties close to 1×10^{-6}, although the estimate was subsequently demonstrated to be biased by an unrecognized systematic effect in the measurement of the lattice parameter of silicon [105]. However, that experiment was a landmark and subsequent work (see, *e.g.* [106-109]) followed the same basic principles of X-ray interferometry coupled with optical interferometry which allowed the measurement of the lattice parameter with the required accuracy. For a description of the XRCD method, see [110-112].

The alternative way towards a kilogram defined in terms of a fundamental constant was invented by Brian Kibble, a NPL scientist, and used in conjunction with the Josephson and the quantum Hall effects to determine the Planck constant with unprecedented accuracy [113]. The experiment compares using the *Kibble balance*, either in two distinct stages or simultaneously, an electromagnetic force and a mechanical force. The former is measured, by means of the Josephson and the quantum Hall effects, in terms of the Planck constant h, the latter in terms of the kilogram, the second and the metre.

$$m = h\left(\frac{bf^2}{4}\right)\frac{1}{gv}\,,$$

where b and f are a dimensionless geometrical term and a frequency depending on the specific experimental setup, g is the acceleration due to gravity and v is the velocity with which the beam of the balance moves under the effect of the mass on its pan [114]. Therefore the mass unit can be obtained by the experiment in terms of the Planck constant h, the second and the metre. For detailed accounts of the method, see [115-117]. For a historical review on the measurement of the Planck constant, see [118].

After the initial NPL measurement, more accurate determinations were published [119-122] with uncertainties comparable to that obtained with the XRCD method.

Most of the determinations of the Boltzmann constant k adopted the method of the acoustic resonator pioneered by Moldover [123]. In it, the speed of sound u is measured in terms of the Boltzmann constant k and the temperature T according to

$$u^2 = \frac{\gamma k T}{m}\,,$$

where $\gamma = 5/3$ for an ideal monatomic gas is the ratio of the gas heat capacities at constant pressure and volume, respectively, and m is the molecular mass of the gas [124]. Other techniques exist, for example the dielectric-constant gas thermometry [125]. For exhaustive descriptions of the methods and of the experimental results about the redefinition of the kelvin in terms of k, see [126, 127].

The draft *mises en pratique* for the realizations of the kilogram, the mole and the kelvin [114, 124, 128] describe how the experiments that were adopted to determine the relevant constants can be used to realize the units.

The CODATA recommended value of the elementary charge [129] is not the result of experimental determination, but is rather obtained from the expression

$$e^2 = \frac{2h\alpha}{\mu_0 c} = 2h\alpha\epsilon_0 c_0.$$

9. – Developments 2006–2018

The revision of the SI was not adopted by the 2007 General Conference, as in the hopes of the proponents, but the thrust toward the revision was ensured by Recommendation 12 [130]. The following period of some years saw an intensification of the debate about the new comprehensive proposal, as well as an acceleration in the efforts to improve the accuracy in the determination of the defining constants. Both in the literature and within the Consultative Committees the debate was alive. The reports of the CCs are particularly useful to trace the evolution of experiments and thoughts which eventually led to the System as presented to the 26th General Conference.

Two distinct attitudes can broadly be found among the protagonists of the saga: those who rejected the innovation, for various reasons, and those who welcomed it and actively tried to contribute to improving it. As expected, the strongest resistances tended to come from those areas which would be, or would be perceived as being negatively affected by the proposed changes, essentially the communities of mass metrology and of chemical metrology. Electrical metrologists were obviously in favour, as well as, although with less emphasis, thermometrists. Those communities which were not affected except for a cosmetic update to the definition of the corresponding units, *i.e.*, time and frequency, length metrology and photometry and radiometry, had of course no reason to react significantly.

Much of the debate was about how many and which constants should be adopted. This debate had two aspects. On the one hand, it should be ensured that the chosen constants were those necessary and sufficient to constitute a set neither insufficient nor over-constrained yet capable of defining all the SI units [95, 131]. On the other hand, advantages and disadvantages of specific sets were debated. Not all chemists did accept the break of the link between the kilogram and the mole, and the uncertainty associated with the molar mass of carbon 12 [132-135], despite the attempts to persuade the community of the advantages of the new definition of the mole [136, 137]. Some alternative sets were also proposed for consideration [95, 132, 133].

Many were worried by the obscurity of the definition of the kilogram, unit of mass, in terms of the Planck constant, an action. This obscurity was used as a reason in favour of a definition based on the mass of an elementary entity such as an atom [138]. The fact that the set of defining quantities is different from the set of quantities to which the defined units belong was perceived as a general weakness concerning the accessibility of the definitions. At the 12th CCM meeting in 2010 [139] I suggested to re-word the definitions, not only that of the kilogram, in such a way that each unit be defined in terms of a monomial expression of the defining constants having the same dimension as the defined unit. The CCU President was attending, and the proposal was well received and eventually (partially) adopted in the final format of the definitions.

However, the main concern of (a minor part of) the community of mass metrologists was about the additional realization uncertainty attached to the kilogram [140], in spite of the demonstration [95] that this additional uncertainty component would be essentially harmless.

The CCM had its 10th meeting in March 2007 [141]. Its main outcome was a better specification of the conditions set in the 2005 recommendation. The subsequent meetings were devoted to the intense work concerning the technical issues related to the practical implementation of the redefinition of the kilogram. The 12th meeting of 2010 [139] issued two Recommendations. The first was a further refinement of the 2005 recommendation, and the second was a request to the CIPM to authorize using the International Prototype for a final verification intended to assess the consistency of mass measurements. This was approved by the CIPM at its 102nd meeting in 2013, together with the CCM Recommendation G1 (2013), a further refinement of the 2005 Recommendation.

In the meantime the CCU had met in 2007, 2009 and 2010. During this last meeting, it was recognized that the conditions established by the CCM still were not met. Nonetheless, it was decided to keep the issue on the table of the General Conference. Therefore, a draft Recommendation was prepared for the CIPM, in which the general structure of the new SI was outlined. This became eventually the Resolution 1 of the 24th CGPM (2011) [142]. The CCU did not meet until 2013 [37], in view of the 25th General Conference of 2014. Consensus was general that neither the conditions on the defining constants were met, nor the practical provisions such as the *mises en pratique* and the new SI Brochure were complete and the revision of the SI could not take place. Therefore, the General Conference issued its Resolution 1 of the 25th CGPM (2014) [143], encouraging to continue the effort. For an overview of the progress until 2014, see [144].

The activity on practical aspect continued in the meantime.

The CODATA had published two adjustments [145,57] showing reduction of the uncertainties in data from the Kibble balances and the XRCD and still inconsistency between the two experiments. In practice an order of magnitude had been gained from 2005, with a relative discrepancy more or less unchanged. The fears that the CCM conditions be too tight were materializing. The Boltzmann constant was in a reassuring situation.

The BIPM was working hard on at least two directions. One was the establishment of an ensemble of mass standards intended to facilitate the dissemination of the mass unit [144], the other was the extraordinary calibration decided in 2010 by the CCM. The calibration involved in a first stage the International Prototype and its *témoins* [19], and subsequently the ten working standards of the BIPM, the so-called mass unit "as maintained" [20]. Both stages gave surprising results. The first demonstrated that the trend shown by the Prototype until the third Verification had stopped, as no further drift was observed with respect to the copies. The second brought to light a mass loss in the BIPM working standards, probably due to anomalous wear caused by a defective balance. This loss impacted on all the calibrations of national prototypes carried out at the BIPM since the year 2003, implying an overestimation of the masses of the standards amounting in the worst cases to as much as 35 μg. Corrected certificates were sent to the involved NMIs. The decision to use a last time the Prototype was decidedly a good decision.

The extraordinary calibration was preliminary to a comparison among those NMIs capable to realize the unit in terms of h. This was carried out during 2016 using stainless-steel kilograms as travelling standards and the results were consistent within the expected range of about 10 μg [146]. In addition, the weighted mean of the results was in agreement with calibrations based on the International Prototype within 1 μg, a datum reassuring about continuity in the definition.

The kilogram, the main obstacle to the revision, seemed in good shape. The CCM at its 16th meeting in 2017 [147] issued the Recommendation G 1 (2017), in practice the green light to the redefinition of the SI, although the most recent experimental data still showed inconsistency, mostly due to the extremely small uncertainties declared [108,148]. The 2017 final CODATA adjustment [149] resolved the issue by appropriately increasing the uncertainties. As to the other units, they were not representing a problem. An overview of the activity of the CCM and the mass community in general related to the redefinition of the SI can be found in [150].

Everything seemed ready for the 23rd CCU meeting in September 2017 [151]. The key topic was the consistency of the experimental data for h. The NIST presented a working document, subsequently published [152], in which the data were analyzed and the conclusion was that they were consistent. The PTB presented documents in which, based on the same data, dangerous inconsistencies were found. I was invited as an unbiased expert and my conclusion was that the original data were not consistent, yet with the moderate increase of uncertainties adopted by the CODATA they became consistent and usable for the revision of the SI.

Eventually the CCU agreed that the time had come for the revision of the SI in 2018.

10. – Attempt to solve the issue of dimensionless quantities

The issue of dimensionless quantities, addressed in sect. **4**˙5, is certainly not prominent in the revision of the SI, yet it attracted me for several reasons and, considering both the simultaneity with the revision proposals and the scientific stature of the protagonists, I feel appropriate to discuss it here, as one of the areas in which the new SI might be improved sooner or later.

At the 21st meeting of the CCU [37] in 2013 Peter Mohr, NIST and IUPAP, presented a note as a working document, in which he came to the provocative conclusion that, according to the SI rules,

$$2\pi\,\mathrm{rad\,s^{-1}} = 1\,\mathrm{cycle\,s^{-1}},\ \text{or}\ 2\pi\,\mathrm{s^{-1}} = 1\,\mathrm{s^{-1}},\ \textit{i.e.}\ 2\pi = 1,$$

as already noted previously [153]. His NIST colleague Bill Phillips, Nobel laureate, presented a companion note concerning analogous ambiguities in random phenomena. It happens that it is common to improperly use the unit hertz also for countings, *i.e.*, events (such as decays) per second. This is justified by the fact that the same counter can equally well count decays per second or cycles per second, depending on the module connected at its input. In cavity quantum electrodynamics (CQED) several expressions involve both decay rates and frequencies, with great potential for confusion.

All the pitfalls presented above can of course be made innocuous by solid knowledge, sound judgement and proper handling of the dimensions involved. However, this is not always the case (and instrument displays do not help, in that counts are often displayed in hertz). In any case, there remains a problem when a computer is involved in data processing. In conclusion, errors by a factor 2π or $(2\pi)^2$ can and, according to the presenters' experience, do occur just because of the ambiguity in the SI concerning radian and steradian.

The president of the CCU, Ian Mills, considered in a further note the ambiguities and the loss of information arising by the strict application of the SI rules in chemistry. He made the example of an atmospheric pollutant, present in 5000 molecules per cubic centimetre, which would be expressed in the SI as $5000\,\mathrm{cm}^{-3}$. The corresponding molar concentration is $\approx 8.022 \times 10^{-23}\,\mathrm{mol/dm}^3$. Both expressions are much less intuitive than the one usually adopted by atmospheric chemists, explicitly mentioning the entity (molecules in this case). He wondered whether it would be appropriate and useful to allow usage of (dimensionless) pseudo-units, such as molecules or, more generally, entities. This enhanced flexibility in the SI would be more adherent to common practice in many sectors of science.

These ideas were embodied in two papers [154, 155]. In the first, the proposal is to make the radian a base unit, thus defining a new dimension, angle. This was not a new idea, as it had surfaced periodically in the literature with various fortunes [153, 156-163]. If the radian is a base unit, obviously ambiguities and potential for errors of the kind described above are removed, and several quantities, including fundamental constants, now having the same units would be clearly distinguished by the different units. For example, angular momentum would have unit joule second per radian, J s/rad. Some precaution must be adopted. For example, in mathematical functions involving angles, such as the complex exponential function $e^{\mathrm{i}\theta}$, connected to angles by the Euler formula $e^{\mathrm{i}\theta} = \sin(\theta) + \mathrm{i}\cos(\theta)$, the argument must be a real number, *i.e.*, dimensionless, because of the power expansion $e^x = 1 + x + \frac{x^2}{2} + \ldots$ in which all terms must have the same dimension. The remedy suggested is to take simply the numerical value of the angle. A conclusion of the paper, with wide repercussions on the expression of many units for quantities, including fundamental constants, is that, since Hz = cyl/s, where cyl is a symbol for cycle, Hz $= 2\pi\,\mathrm{rad\,s}^{-1}$. Now, both the radian and the hertz cannot be coherent SI units (a coherent derived unit is such that all the numerical factors in its expression in terms of base units are equal to one). Since the radian is considered a coherent unit, "one conclusion that is not optional is that the unit hertz cannot be regarded as a coherent unit of the SI, in contrast to its designation in the current form of the SI, where cycles are ignored and Hz may be replaced by s^{-1}" [154].

A provocative proposal came also with respect to phenomena involving countings, such as a number of events in a given time or the number of entities in a given volume, or mass. The proposal was to explicitly mention the entities and the events under consideration, thus introducing the pseudo-units advocated by Mills. In this way, for example, the Avogadro constant N_A would have unit $\mathrm{ent\,mol}^{-1}$, where ent stands for entity.

Mill's paper [155] was more or less on the same line, explicitly supporting the need to give a dimension to angles.

The reactions were numerous, as easily imaginable. Leonard [164] interpreted the paper as an outrage to the SI and corrected those which in his opinion were truly errors. Replies from the proponents [165, 166] restated their positions. Quincey [167] and

Quincey & Brown [168] review the various ways of handling angles in the SI and, in the case of adoption of a dimension angle, suggest the introduction of a dimensional constant to modify accordingly the expressions of the relevant physical quantities, a suggestion already proposed by Brownstein [153].

In the meantime, a Working Group on Angles and Dimensionless Quantities in the SI (CCU-WGADQ) had been established under the chair of a PTB staff (the President of the CCU had also changed, and was the President of the PTB) with the task of examining the issues arising with these quantities, to report to the CCU on possible solutions and to suggest appropriate text to the drafting team of the 9th edition of the SI Brochure. The WG met at the BIPM in 2015 and no consensus was reached. Two versions were submitted, one containing the ideas put forward by Mohr and Phillips, and supported by a number of members of the WG (including the author of this paper), another with a German proposal [169] to formally introduce in the SI a further quantity Number and a corresponding dimension **Z**, having the status of the neutral element of the group of SI dimensions, and not belonging to base quantities. Neither of the two proposals was implemented in the 9th edition of the SI Brochure. I feel, with many other colleagues, that not profiting of this major revision of the SI to give it a more solid basis by a modification of the status of radian and steradian was a missed occasion. I do hope that in a not-too-far future this change will be implemented.

11. – The third-millennium international system

The System eventually sanctioned by the 26th CGPM is the result of all the debates of the previous years. Therefore, the choice was for giving the list of the defining invariants, with their SI values having no uncertainty. In addition, since the decision had been taken to keep the distinction between base and derived units for historical reasons, the base units are defined by means of an evolution of the explicit-constant style.

A good idea of the complexity of the path that eventually led to the revision is given by a roadmap jointly prepared by the CCM and CCU Presidents, whose essentials are reproduced in fig. 7.

The 9th edition of the SI Brochure describes in full detail the structure of the new System, as well as the definitions of the base quantities and the *mises en pratique*. Most of the material below is taken from that Brochure, still a draft at the time of writing [170].

The International System of Units, the SI, is the system of units in which

- the unperturbed ground state hyperfine transition frequency of the cæsium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is 9 192 631 770 Hz,
- the speed of light in vacuum c is 299 792 458 m/s,
- the Planck constant h is $6.626\,070\,15 \times 10^{-34}$ J s,
- the elementary charge e is $1.602\,176\,634 \times 10^{-19}$ C,
- the Boltzmann constant k is $1.380\,649 \times 10^{-23}$ J/K,
- the Avogadro constant N_{A} is $6.022\,140\,76 \times 10^{23}\ \mathrm{mol}^{-1}$,
- the luminous efficacy of monochromatic radiation of frequency 540×10^{12} hertz K_{cd} is 683 lm/W,

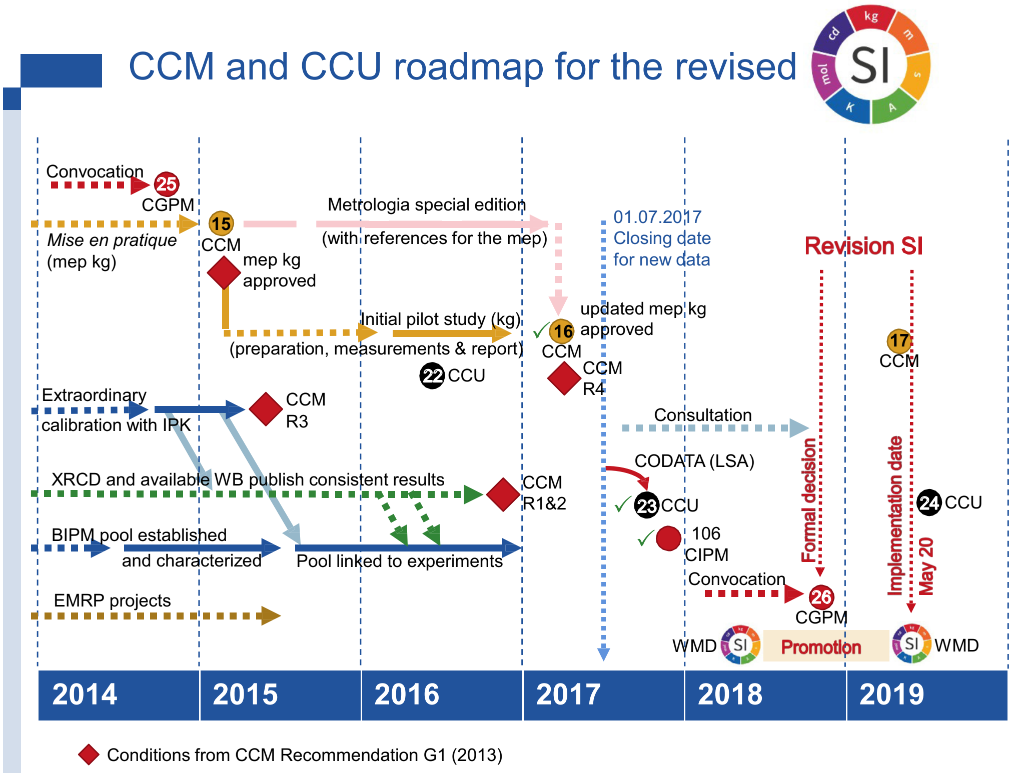

Fig. 7. – SI Roadmap 2014-2019. R1 to R4 refer to checks of consistency, uncertainty, traceability and validation, respectively, as established by the CCM Recommendations (courtesy BIPM).

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to $\mathrm{Hz} = \mathrm{s}^{-1}$, $\mathrm{J} = \mathrm{m}^2\,\mathrm{kg}\,\mathrm{s}^{-2}$, $\mathrm{C} = \mathrm{A\,s}$, $\mathrm{lm} = \mathrm{cd\,m}^2\,\mathrm{m}^{-2} = \mathrm{cd\,sr}$, and $\mathrm{W} = \mathrm{m}^2\,\mathrm{kg\,s}^{-3}$.

The values of the constants are those determined with the 2017 special CODATA adjustment [149]. For a list of the most recent input data for this adjustment, see [171].

The second

The second, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$, the unperturbed ground-state hyperfine transition frequency of the cæsium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

This definition implies the exact relation $\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770\,\mathrm{Hz}$. Inverting this relation gives an expression for the unit second in terms of the value of the defining constant $\Delta\nu_{\mathrm{Cs}}$

$$1\,\mathrm{Hz} = \frac{\Delta\nu_{\mathrm{Cs}}}{9\,192\,631\,770} \quad \text{or} \quad 1\,\mathrm{s} = \frac{9\,192\,631\,770}{\Delta\nu_{\mathrm{Cs}}}\,. \tag{16}$$

The metre

The metre, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit $\mathrm{m\,s^{-1}}$, where the second is defined in terms of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$.

This definition implies the exact relation $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$. Inverting this relation gives an exact expression for the metre in terms of the defining constants c and $\Delta\nu_{\mathrm{Cs}}$

$$(17)\qquad 1\,\mathrm{m} = \Bigl(\frac{c}{299\,792\,458}\quad \mathrm{s} = \frac{9\,192\,631\,770}{299\,792\,458}\frac{c}{\Delta\nu_{\mathrm{Cs}}} \approx 30.663\,319\frac{c}{\Delta\nu_{\mathrm{Cs}}}\,.$$

The kilogram

The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit J s, which is equal to $\mathrm{kg\,m^2\,s^{-1}}$, where the metre and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$.

This definition implies the exact relation $h = 6.626\,070\,15\times10^{-34}\,\mathrm{kg\,m^2\,s^{-1}}$. Inverting this relation gives an exact expression for the kilogram in terms of the three defining constants h, $\Delta\nu_{\mathrm{Cs}}$ and c

$$(18)\qquad 1\,\mathrm{kg} = \left(\frac{h}{6.626\,070\,15\times 10^{-34}}\right)\,\mathrm{m^{-2}\,s},$$

which is equal to

$$(19)\quad 1\,\mathrm{kg} = \frac{299\,792\,458^2}{6.626\,070\,15\times10^{-34} 9\,192\,631\,770}\frac{h\Delta\nu_{\mathrm{Cs}}}{c^2} \approx 1.475\,521\,4\times10^{40}\frac{h\Delta\nu_{\mathrm{Cs}}}{c^2}\,.$$

The ampere

The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \times 10^{-19}$ when expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$.

This definition implies the exact relation $e = 1.602\,176\,634\times10^{-19}\,\mathrm{A\,s}$. Inverting this relation gives an exact expression for the unit ampere in terms of the defining constants e and $\Delta\nu_{\mathrm{Cs}}$

$$(20)\qquad 1\,\mathrm{A} = \left(\frac{e}{1.602\,176\,634\times10^{-19}}\right)\,\mathrm{s^{-1}},$$

which is equal to

$$(21)\quad 1\,\mathrm{A} = \frac{1}{(9\,192\,631\,770)(1.602\,176\,634\times10^{-19})}\Delta\nu_{\mathrm{Cs}}e \approx 6.789\,687\times10^8\Delta\nu_{\mathrm{Cs}}e.$$

The kelvin

The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be $1.380\,649 \times 10^{-23}$ when expressed in the unit $\mathrm{J\,K^{-1}}$, which is equal to $\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{\mathrm{Cs}}$.

This definition implies the exact relation $k = 1.380\,649 \times 10^{-23}\,\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$. Inverting this relation gives an exact expression for the kelvin in terms of the defining constants k, h and $\Delta\nu_{\mathrm{Cs}}$

$$1\,\mathrm{K} = \left(\frac{1.380\,649}{k}\right) \times 10^{-23}\,\mathrm{kg\,m^2\,s^{-2}}, \tag{22}$$

which is equal to

$$1\,\mathrm{K} = \frac{1.380\,649 \times 10^{-23}}{(6.626\,070\,15 \times 10^{-34})(9\,192\,631\,770)} \frac{\Delta\nu_{\mathrm{Cs}} h}{k} \approx 2.266\,665\,3 \frac{\Delta\nu_{\mathrm{Cs}} h}{k}\,. \tag{23}$$

The mole

The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.022\,140\,76 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_{A}, when expressed in the unit $\mathrm{mol^{-1}}$ and is called the Avogadro number.

The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

This definition implies the exact relation $N_{\mathrm{A}} = 6.022\,140\,76 \times 10^{23}\,\mathrm{mol^{-1}}$. Inverting this relation gives an exact expression for the mole in terms of the defining constant N_{A}

$$1\,\mathrm{mol} = \left(\frac{6.022\,140\,76 \times 10^{23}}{N_{\mathrm{A}}}\right). \tag{24}$$

It is worth noting that the format of this definition of the mole is at variance with respect to those of the other units. The form above is a compromise between an even more diverging formulation presented by the CCQM and following extensive debate within the International Union of Pure and Applied Chemistry, IUPAC, and the standard formulation.

The candela

The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}\,\mathrm{Hz}$, K_{cd}, to be 683 when expressed in the unit $\mathrm{lm\,W^{-1}}$, which is equal to $\mathrm{cd\,sr\,W^{-1}}$, or $\mathrm{cd\,sr\,kg^{-1}\,m^{-2}\,s^3}$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{\mathrm{Cs}}$.

This definition implies the exact relation $K_{\mathrm{cd}} = 683\,\mathrm{cd\,sr\,kg^{-1}\,m^{-2}\,s^3}$ for monochromatic radiation of frequency $\nu = 540 \times 10^{12}\,\mathrm{Hz}$. Inverting this relation gives an exact

Fig. 8. – The logo of the new SI (courtesy BIPM).

expression for the candela in terms of the defining constants K_{cd}, h and $\Delta\nu_{\mathrm{Cs}}$

$$1\,\mathrm{cd} = \left(\frac{K_{\mathrm{cd}}}{683}\right)\,\mathrm{kg\,m^2\,s^{-3}\,sr^{-1}}, \tag{25}$$

which is equal to

$$\begin{aligned} 1\,\mathrm{cd} &= \frac{1}{(6.626\,070\,15\times10^{-34})(9\,192\,631\,770)^2 683}\,(\Delta\nu_{\mathrm{Cs}})^2\,hK_{\mathrm{cd}} \\ &\approx 2.614\,830\times10^{10}\,(\Delta\nu_{\mathrm{Cs}})^2\,hK_{\mathrm{cd}}. \end{aligned} \tag{26}$$

The logo of the new SI (fig. 8) shows schematically the seven base units and the associated defining constants.

12. – Impact

In general, uncertainty is a measure of the state of knowledge. As such, it cannot magically disappear (or increase) unless the state of knowledge has changed accordingly. However, the (overall) state of knowledge about physical constants and units can only change if new information is provided by, say, new measurements or discoveries, which is not the case in the revision of the SI. Yet, there is no trick in the revision, as well as there was no trick when the speed of light c_0 was promoted from uncertain to exact. In both cases, uncertainty did not disappear, rather it was distributed differently from before. We trust that each physical constants has a unique value, and we are allowed to fix its numerical value in the SI by choosing a suitable real number exactly known at the expenses, so to say, of the corresponding (SI) unit. So, in a sense, uncertainty is transferred from the physical constants h, k, e and N_{A} to the artefacts traditionally used to realize the unit, *i.e.*, the international prototype of the kilogram and the triple point of water, and to some of the previously exact constants, which acquire an uncertainty. This is the case of the magnetic constant μ_0 (and ϵ_0 and Z_0, which are related to it), and of the molar mass constant M_u. This aspect has been variously questioned [135,172] but overall the re-distribution of uncertainty determined by the revision of the SI is more rational and corresponds better to our understanding of nature. See also [95].

The revision of the SI has been beneficial to different extents for the various fields of metrology, but some price has been paid. In the following subsections I will examine the impact of the revised SI on the various concerned fields. The drafts of the *mises en pratique* for the various quantities give useful information. They are cited separately in the following sections.

12˙1. *Mass metrology.* – The Prototype $\mathfrak{K}$, at the time of the redefinition, still has mass 1 kg, but now with a relative uncertainty of 1×10^{-8}, *i.e.*, a standard uncertainty $u(m(\mathfrak{K})) = 10\,\mu\mathrm{g}$. Roughly speaking, a relative uncertainty not larger than 1×10^{-8} is acquired by all the mass standards traceable to $\mathfrak{K}$. This component is originated by the realization of the unit, a step which did not exist in the previous SI, and affects both the uncertainty of mass estimates and the covariance $\mathrm{Cov}(m_i, m_j)$ between any two of them [95]. Therefore, it cancels in any comparison, and its effect is null within the domain of mass metrology. Of course, in the case of measurements involving other quantities beside mass, the component must be properly taken into account.

As to the Prototype, it might be interesting in the future to follow the evolution of its mass with respect to the Planck constant, to check whether the trend showed so far is confirmed. It is unlikely that it will have a further prominent role in metrology and science, so that it will at last enjoy a well-deserved retirement after almost 150 years of glorious career.

As to traceability and dissemination, the path changes considerably. Individual NMIs in possession of one of the means contemplated in the *mise en pratique* [114] of the kilogram, *i.e.*, the Kibble balance or the XRCD experiment, can disseminate the mass unit with an uncertainty corresponding to that demonstrated by a successful participation in suitable key comparisons (as the one mentioned in [146]). However, due to the data for h subsequent to the comparison, indicating a worsening of the consistency level, a transitory phase during which dissemination starts from a consensus value, rather than from individual values, was decided [173]. A focus issue of the Journal *Metrologia* is devoted to the Realization, maintenance and dissemination of the kilogram in the revised SI [174].

The next generation of Kibble balances will probably enable the direct calibration of masses having values different from the unit, 1 kg [175], which will deeply change the face of mass metrology, currently tied to the dissemination from the unit. Further and perhaps better, or simplified Kibble balances are likely to be developed to ensure continuity and improve ease of use [176-178].

12˙2. *Electrical metrology.* – Electrical metrology greatly benefits from the SI revision. The situation that started in 1990, when the representations of the ohm and the volt in terms of the Josephson and von Klitzing constants were introduced, and these constants were given conventional exact values (see sect. **4**˙2), was satisfactory from a practical viewpoint, but left much to be desired from the wiewpoint of the general consistency of the system. In a sense, current electrical measurements were not traceable to the SI. The role of the conventional exact values $K_{\text{J-90}}$ and $R_{\text{K-90}}$ is terminated with the revision of the SI, and they play no further role in the revised system. K_{J} and R_{K} have no longer conventional counterparts, and the volt and the ohm are realized from the Josephson and quantum Hall effect, respectively, where the values of the two constants are now exact. There is a price to pay, however, in terms of a discontinuity with respect to the past. The value of K_{J} is smaller than $K_{\text{J-90}}$ by 10.67×10^{-8} and the value of R_{K} is larger than $R_{\text{K-90}}$ by 17.79×10^{-9}. As a consequence, both the new SI volt and the new SI ohm are smaller by the same relative amounts with respect to the previous units. This further implies that the same voltage measured in the new SI has a numerical value that is greater than that measured in the old SI, and that the same resistance measured in the new SI has a numerical value that is greater than that measured in the old SI [179]. All the measurements of voltage-related and resistance-related quantities are of course affected in the same way, respectively.

The magnetic constant μ_0 is no longer exact. This impairs in principle the exact conversion between the SI and the electromagnetic systems, and in practice makes them obsolete [180, 181]. In practice, the value at the moment of the redefinition changes by the fractional amount of 2×10^{-10}, with a relative uncertainty equal to 2.3×10^{-10}, a change negligible under every respect.

12˙3. *Physical constants.* – This is the field where the impact is more evident. The Planck constant h, the Boltzmann constant k, the Avogadro constant N_{A} and the elementary charge e become exact at the expenses of the mass of the Prototype $m(\mathfrak{K})$ (see sect. **12**˙1), the temperature of the triple point of water T_{tpw} (see sect. **12**˙4), the molar mass constant M_{u} (see sect. **12**˙5) and the magnetic constant μ_0 (see sect. **12**˙2). The von Klitzing and Josephson constants become exact, as well as the molar gas constant $R = kN_{\text{A}}$, the Faraday constant $F = eN_{\text{A}}$, the Stefan-Boltzmann constant σ and many others. For an exhaustive list, see [76]. The SI revision is a true revolution for fundamental constants.

12˙4. *Thermometry.* – The impact on thermometry is limited to thermodynamic temperature. Therefore, practical measurements, based on the ITS-90, are unaffected. The temperature of the triple point of water, T_{tpw}, does change, due to the definition, by a meaningless fractional amount of 2×10^{-8}, mostly due to rounding, with a standard uncertainty $u(T_{\text{tpw}}) = 0.1\,\text{mK}$. Prior to the revision T_{tpw} had no uncertainty and its reproducibility was evaluated to about $50\,\mu\text{K}$. With the new definition, the kelvin can be realized at any point of the temperature range, using one of the techniques described in the *mise en pratique* [124], with advantages in terms of uncertainty with respect to the situation prior to the revision.

12˙5. *Metrology in chemistry.* – Fixing the Avogadro constant is beneficial to chemistry in that many constants widely used become exact. The price is that the molar mass constant is no longer exact, which is claimed to introduce a fundamental incompatibility in stoichiometric equations [182]. In practice $M(^{12}\text{C})$ changes by effect of the revision by the fractional amount 3.7×10^{-10}, with a relative uncertainty equal to 4.5×10^{-10}.

13. – Conclusion

I feel that the best possible conclusion is with a quotation from the same James Clerk Maxwell cited at the beginning of this paper. His visionary dream is now much closer to reality after the revision of the SI that I tried to describe.

> *Yet, after all, the dimensions of our earth and its time of rotation, though, relatively to our present means of comparison, very permanent, are not so by any physical necessity. The earth might contract by cooling, or it might be enlarged by a layer of meteorites falling on it, or its rate of revolution might slowly slacken, and yet it would continue to be as much a planet as before. But a molecule, say of hydrogen, if either its mass or its time of vibration were to be altered in the least, would no longer be a molecule of hydrogen. If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wave-length, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules. When we find that here, and in the starry heavens, there are innumerable multitudes of little bodies of exactly the same mass, so many, and no more, to the grain, and vibrating in exactly the same time, so many times, and no more, in a second, and when we reflect that no power in nature can now alter in the least either the mass or the period of any one of them, we seem to have advanced along the path of natural knowledge to one of those points at which we must accept the guidance of that faith by which we understand that "that which is seen was not made of things which do appear."*
>
> James Clark Maxwell ([96], p. 225).

* * *

I gratefully acknowledge the many colleagues at INRiM and in several Consultative Committees whith which I exchanged ideas on the vast subject of the SI. Franco Cabiati, INRiM and member of the CODATA TGFC, deserves special thanks for many stimulating suggestions he made during the long cooperation we had on the topic of the SI. My warmest thanks go to Prof. Alessandro Bettini, University of Padova, for proposing me to write this paper. His proposal made me aware of the immensity of my ignorance compared to my knowledge, and gave me the privilege of living this fascinating albeit sometimes painful adventure.

Glossary of principal symbols and acronyms

$A_{\mathrm{r}}(\mathrm{X})$ relative atomic mass of the entity X

BIPM Bureau international des poids et mesures

c_0 speed of light in vacuum

CCE Comité consultatif d'électricité. Consultative Committee for Electricity and Magnetism (former denomination of the CCEM)

CCEM Comité consultatif d'électricité et magnétisme. Consultative Committee for Electricity and Magnetism

CCM Comité consultatif pour la masse et les grandeurs apparentées. Consultative Committee for Mass and Related Quantities

CCM WGAC Working Group on the Avogadro Constant of the Consultative Committee for Mass and Related Quantities (CCM)

CCM WGSI *Ad hoc* Working Group on changes to the SI of the Consultative Committee for Mass and Related Quantities (CCM)

CCPR Comité consultatif de photométrie et radiométrie. Consultative Committee for Photometry and Radiometry

CCQM Comité consultatif pour la quantité de matière. Consultative Committee for Amount of Substance: Metrology in Chemistry and Biology

CCT Comité consultatif de thermométrie. Consultative Committee for Thermometry

CCU Comité consultatif des unités. Consultative Committee for Units

CCU WGADQ CCU Working Group on Angles and Dimensionless Quantities in the SI of the Consultative Committee for Units (CCU)

CGPM Conférence générale des poids et mesures. General Conference on Weights and Measures

CIPM Comité internationsl des poids et mesures. International Committee of Weights and Measures

CODATA Committee on Data for Science and Technology

CODATA TGFC CODATA Task Group on Fundamental Constants

$\mathrm{Cov}(X_i, X_j)$ covariance for two random variables X_i and X_j

CQED Cavity Quantum Electrodynamics

EURAMET The European Association of National Metrology Institutes

IAEA International Atomic Energy Agency

IEC International Electrotechnical Commission

IENGF Istituto elettrotecnico nazionale "Galileo Ferraris"

IMGC Istituto di metrologia "Gustavo Colonnetti"

INRiM Istituto Nazionale di Ricerca Metrologica

IPTS-68 International Practical Temperature Scale of 1968

ITS-90 International Temperature Scale of 1990

IUPAC International Union of Pure and Applied Chemistry

IUPAP International Union of Pure and Applied Physics

$\mathfrak{K}$ The International Prototype of the kilogram

K_{J} Josephson constant $K_{\mathrm{J}} = 2e/h$

$K_{\mathrm{J\text{-}90}}$ conventional value of Josephson constant

KCDB The BIPM key comparison database

$M(^{12}\mathrm{C})$ molar mass of $^{12}\mathrm{C}$

M_{u} molar mass constant

N_{A} Avogadro constant

NBS National Bureau of Standards (former denomination of NIST)

NIST National Institute of Standards and Technology

NMI National Metrology Institute

NPL National Physical Laboratory

PTB Physikalisch-Technische Bundesanstalt

QHE Quantum Hall effect

$r(x_i, x_j)$ correlation coefficient associated with the estimates x_i and x_j of the input quantities X_i and X_j

R_∞ Rydberg constant

R_{K} von Klitzing constant $R_{\mathrm{K}} = h/e^2$

$R_{\mathrm{K\text{-}90}}$ conventional value of von Klitzing constant

T_{tpw} temperature of the triple point of water

VSMOW Vienna Standard Mean Ocean Water

XRCD X-Ray Crystal Density

Z_0 characteristic impedance of free space

α fine-structure constant

ϵ_0 electric constant, permittivity of free space

μ_0 magnetic constant, permeability of free space

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