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Key words: Uncertainty, nonlinearity, estimate, decibel

1 Expression of quantities in decibel

When dealing with a *power-related quantity*, P, one is often interested in the ratio P_{dB} of its value to a reference value P_0 rather than to its SI value. The quantity P_{dB} is expressed in decibel (dB) as the decimal logarithmic ratio of P with respect to P_0:

$$P_{\mathrm{dB}} = 10 \log_{10} \frac{P}{P_0} \tag{1}$$

$$P = P_0 10^{P_{\mathrm{dB}}}. \tag{2}$$

Equations (1) and (2) can be considered as measurement models, whose measurands P_{dB} and P are function of the input quantities P and P_{dB}, respectively (P_0 is a constant).

2 Estimate and uncertainty

The JCGM 100 [1] approach to uncertainty evaluation is based essentially on Gauss' law of propagation of variances, normally implying the linearisation of the measurement model around the estimates of the input quantities. It is acknowledged that non-linearities sufficiently strong in neighborhoods of the input estimates comparable with the magnitudes of the associated standard uncertainties might affect

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the evaluated uncertainty. What is not acknowledged is that non-linearities (as well as asymmetrically distributed input values) *can also bias the measurand estimate*. JCGM 101 [2] automatically takes these effects into account by assigning state-of-knowledge probability distributions for the input quantities and propagating them through the measurement model to produce a probability distribution for the possible values of the measurand. JCGM 100, on the contrary, does not explicitly discuss this topic, and only considers a correction to the measurand estimate in an example given in an Annex (see [1], F.2.4.4). Thus, in general, both the estimates and the standard uncertainties provided by [1] differ from those of [2], the latter document being considered as the gold standard.

The strong non-linearity of models (1) and (2) suggests that the JCGM 100 approach might be inadequate.

Indeed, in GUM-6 [3] it is written "…it is not recommended to perform uncertainty calculations using logarithmic quantities in, for example, decibel. Logarithmic quantities should be converted to the equivalent linear format before performing uncertainty calculations" (JCGM-GUM6).

When P_{dB} has a normal distibution, P has a lognormal one. This assumption appears reasonable when considering the structure of a typical acoustic or radio-frequency power measurement. The signal is given by a number of devices in cascade (transducers, amplifiers, filters, detectors …), each one characterised by a signal gain with an uncertainty. When the signal is expressed in dB, all device gains sum up with the original signal power; in the linear domain, instead, gains are multiplied. It is therefore reasonable that for dB quantities the additive central limit theorem can be applied, and for linear quantities the multiplicative one, resulting in normal and log-normal distributions of the quantity value respectively.

The talk will discuss exact conversion formulae for the distribution parameters, the mean and the standard deviation of power quantities, from the linear to dB representation to the linear one and vice versa, highlighting the bias introduced by the JCGM 100 approach.

References

[1] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Evaluation of measurement data — Guide to the expression of uncertainty in measurement. Joint Committee for Guides in Metrology, JCGM 100:2008, 2008.

[2] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Evaluation of measurement data — Supplement 1 to the "Guide to the expression of uncertainty in measurement" — Propagation of distributions using a Monte Carlo method. Joint Committee for Guides in Metrology, JCGM 101:2008, 2008.

[3] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. Guide to the expression of uncertainty in measurement — Part 6: Developing and using measurement models. Joint Committee for Guides in Metrology, JCGM GUM-6:2020, 2020.