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Giovanni Giorgi and the International System of Units

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1 Introduction

Electrical engineers and technicians face today many professional challenges, but the problem of the choice of the measurement units and their conversion is not one of those. Our colleagues working in mechanics or thermal engineering have to deal with inches and metres, gallons and litres, calories, BTUs and joules. In electricity, everything is smoother: the voltage is always measured in volt, the resistance in ohm; one volt times one ohm gives one watt, and so on. No ambiguities occur, no calculations with weird conversion factors are needed.

Several generations of scientists contributed to create a system of units that we electrical people so much enjoy. If a single name has to be given, however, most metrologists would choose Giovanni Giorgi.

As I write, 120 years have passed since Giorgi's original proposal *Rational units of electromagnetism* [1, 2]. Giorgi's main thesis was that it is possible to express electromagnetic quantities in a (then) new system of units, founded on firm theoretical basis and at the same time practical to use for both physics and engineering. *Giorgi's system*, as the proposal became known, evolved and expanded into the International System of Units (SI) [3, 4], now accepted worldwide.

In 2019, the SI underwent a profound revision. Four of the seven SI base units were redefined in terms of constants that describe the natural world. In particular, the base unit ampere is now defined in terms of the elementary electric charge e. It is therefore worthwhile to discuss Giorgi's proposal in such a new framework.[1]

2 The cgs systems of units

Units for electromagnetic quantities can be derived from the mechanical base units, by considering the Coulomb and Ampere force laws for point charges q_1, q_2 or straight parallel current paths I_1, I_2 at distance d are respectively:

$$F_\mathrm{C} = k_\mathrm{C} \frac{q_1\, q_2}{d^2}$$

$$F_\mathrm{A} = 2k_\mathrm{A} \frac{I_1\, I_2}{d^2}$$

where a choice of the values of the constants k_C and k_A defines the size of the units for the charge q or current I, respectively. The two constants are linked by Maxwell's equations, $k_\mathrm{C}/k_\mathrm{A} = c^2$, where c is the speed of light.

The cgs (centimetre-gram-second) mechanical system of units was extended to electromagnetic phenomena in two main ways:

- the choice $k_\mathrm{C} \equiv 1$ defines the *electrostatic* (cgs-es) system of units;
- the choice $k_\mathrm{A} \equiv 1$ defines the *electromagnetic* (cgs-em) system of units.

The size of the units generated by these two choices is wildly different. Table 1 gives the size of some cgs-es and cgs-em units in terms of the modern SI units, together with their names.[2]

[1] This paper was presented during the dedication ceremony of the IEEE Milestone "Giovanni Giorgi's Contribution to the Rationalized System of Units, 1901-1902", on 15 December 2021. A more comprehensive version will appear in a forthcoming volume in the History Notes Series, edited by the IEEE History Activity Committee, Italy Section.

[2] The names were not universally accepted and in some cases were recognized decades after the system was invented.

Quantity	cgs-es		cgs-em	
Current	statampere	336 pA	abampere	10 A
Voltage	statvolt	300 V	abvolt	10 nV
Resistance	statohm	899 GΩ	abohm	1 nΩ
Capacitance	cm, statfarad	1.1 pF	abfarad	1 GF
Inductance	—	—	abhenry	1 nH

Table 1: Some cgs units for electromagnetic quantities. An approximate conversion to the corresponding SI units is also given.

3 Practical units

Towards the end of the nineteenth century, electrical engineers did not consider the cgs systems of units convenient enough for applications. A new practical "international" system of units was progressively introduced. The unit sizes of the new system were intended to be multiples or submultiples of those of the corresponding cgs units, but were given with specific practical realisation rules. Their definitions were established in a series of international congresses [5]. Of particular interest is the 1893 International Congress of Chicago, by which the practical realisations of Table 2 were introduced.

4 Noncoherence

A problem with the practical units is that, although they are intended to be multiples of cgs units, their definition is in fact independent and introduces a *new* set of natural constants, related to properties of matter. Taking the ohm, for example, it is apparent that the values 14.4521 g and 106.3 cm were carefully chosen to act as a conversion factor, such that the size of the ohm matched an exact decadic multiple (10^9) of the abohm. The choice was made on the basis of the best experiments available at the time: however, when further experimental knowledge became eventually available, and the experimental values were then found to be different, the practical ohm became no more a perfect multiple of the abohm. Over time the two systems drifted apart from each other.

The practical system is *noncoherent*[3] because it is not intended to be used independently, but in connection with the cgs units in the description of electromechanical phenomena. The electromagnetic unit of power $1\,\mathrm{V}\cdot 1\,\mathrm{A}$ is not just a simple decadic multiple of the mechanical unit of power $1\,\mathrm{erg} = 1\,\mathrm{g\,cm^2\,s^{-2}}$; it has an independent definition.

The practical system of units has also an intrinsic noncoherence. The three electrical units volt, ampere, and ohm have independent definitions, hence the relation $1\,\mathrm{V} = 1\,\mathrm{A}\cdot 1\,\Omega$, for the very same reason as above, may be not satisfied after an improvement of the experimental knowledge.

5 Giorgi's proposal

Giorgi's proposal was that any system of units has to be constructed on the basis of three pillars:

Rationalization. The system of units must be *rational*, chosen such that the electromagnetic equations include the factor π (or its multiples, 2π and 4π) only when it makes sense because of the geometry. The cgs-es system is not rational, because the equation for the capacitance of a parallel-plate capacitor, $C = \frac{1}{4\pi}\frac{S}{d}$ includes a factor 4π which is unjustified on the basis of the planar geometry.

> No doubt, the student who meets for the first time equations of this kind, is induced to think that 4π arises mysteriously from the

[3] A system of units is coherent if it is defined in such a way that the equations relating the numerical values of the quantities have the same form, including numerical factors, of the corresponding physical equations relating the quantities.

Quantity	Unit	Definition
Current	ampere	The unvarying current which, when passed through a solution of silver nitrate in water, deposits silver at the rate of 0.001 118 00 grams per second
Voltage	volt	$\frac{1000}{1434}$ of the electromotive force of a Clark cell at a temperature of 15 °C
Resistance	ohm	The resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14.4521 g in mass, of a constant cross-sectional area and of the length of 106.3 cm

Table 2: The international units, as established in 1893.

> most intimate nature of electromagnetic phenomena [...] we ought then to define π electromagnetically, and determine its value by measuring [...] the capacity of a plane condenser. [2]

Four base units. Three base units (for length, mass, time) are necessary to describe the mechanical phenomena. Electromagnetism, being a *new* phenomenon, deserves its own *new* unit of measurement.

The original proposal of Giorgi was in fact to add *two* electromagnetic units, but in a way that satisfies coherence.

Coherence. The definition of the added base unit(s) for electromagnetic phenomena must be chosen in such a way that the unit for electromagnetic energy and the unit for mechanical energy is the same.

> On considering the cross-connections established by the circuit laws, we notice that the fundamental units needed are reduced to a common one for [electromotive force] and magnetic current, and to another for [magnetomotive force] and electric current. Their product must reproduce the mechanical unit of activity [power]. In the limits of this condition, their choice is entirely arbitrary.
>
> If the *watt* is assumed as unit of activity, we have two units ready made, the *volt* and the *ampere*, which satisfy the condition. Let us assume them as fundamental. [...]
>
> From the fundamental set here assumed, a complete system of electric and magnetic units can be deduced. This system is rationalized. [2]

6 The SI, 1960-2019

The Metre Convention acknowledged the extent of Giorgi's proposal, and the metric system incorporated it, but with an important twist: the choice of the ampere as the individual base unit. The definition

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

makes the system both rational and coherent.

The direct realisation of the unit can be performed with the *current balance* (or *ampere balance*) experiment, where the electrodynamic force between two coils is measured by in terms of a weight. Such realisation requires, in addition to the knowledge of the gravitational acceleration g, an extremely careful measurement of the mechanical dimensions of the coils and their positions in space, limiting the uncertainty of the best realisations to a few parts in 10^6.

The uncertainty can be improved by linking the outcomes of two different electromechanical experiments, the *watt balance* or *Kibble balance* [6], where the electromagnetic watt is realized in terms of the mechanical watt, and the *calculable capacitor*, which allows for the realization of the impedance units (farad, ohm, henry).

Such realisations are extremely challenging and expensive. In practice, the large majority of the National Metrology Institutes belonging to the Metre Convention maintained the electrical units by calibrated artifact standards.

7 Quantum standards

The discovery of quantum phenomena in solid-state devices revolutionized electrical metrology. Two effects are of particular relevance:

Josephson effect The Josephson effect occurs in devices made of superconducting tunnel junctions, when irradiated by a microwave radiation at frequency f. In proper conditions, the voltage V on the device is $V = \frac{n f}{K_\mathrm{J}}$, where n is an integer and $K_\mathrm{J} = \frac{2e}{h}$ is the *Josephson constant*, in turn given by the elementary charge e and the Planck constant h.

quantum Hall effect The quantum Hall effect occurs in two-dimensional electron gases, at low temperature and under a strong perpendicular magnetic field. The quantum Hall resistance $R_\mathrm{H} = R_\mathrm{K}/i$, where i is an integer, is a simple fraction of the *von Klitzing constant* $R_\mathrm{K} = \frac{h}{e^2}$. The von Klitzing constant is thus again given by the very same fundamental constants of the Josephson constant, the elementary charge and the Planck constant.

Both the Josephson and the quantum Hall effects are *universal*: they provide a quantized voltage or resistance which has been experimentally confirmed to be independent of the specific devices or physical conditions up to extreme accuracies, down to parts in 10^{16} for the Josephson voltage and to parts in 10^{11} for the quantum Hall resistance.

With these *quantum electrical metrology standards* it is possible to generate voltages and resistance with an extremely high reproducibility, at the level of parts in 10^9 or better.

8 Conventional electrical units

In the 1960-2019 SI the Josephson and von Klitzing constants that allow the reproduction of a quantum voltage and resistance must be experimentally determined in terms of the SI electromagnetic units, hence with their electromechanical realisations.

In 1989, the experimental knowledge at the time [7] was

$$K_\mathrm{J} = 483\,597.9(2)\,\mathrm{GHz\,V^{-1}} \qquad [4\times10^{-7}],$$
$$R_\mathrm{K} = 25\,812.807(5)\,\Omega \qquad [2\times10^{-7}].$$

The relative uncertainties are reported in square brackets. A problem arised: the reproducibility of quantum experiments was two to three orders of magnitude better than the uncertainty contribution due to the knowledge of the constants. Consequently, electrical metrologists were keen to compare their measurements in terms of the reproduced quantum values, instead of the actual electromechanical SI units.

This preference was officially recognized within the Metre Convention, with the introduction of the *conventional units*. The conventional *exact* values $K_{\mathrm{J}-90}$ and $R_{\mathrm{K}-90}$ were introduced [8], by simply dropping the uncertainty of the 1989 determination:

$$K_{\mathrm{J}-90} = 483\,597.9\,\mathrm{GHz\,V^{-1}} \qquad [\mathrm{exact}],$$
$$R_{\mathrm{K}-90} = 25\,812.807\,\Omega \qquad [\mathrm{exact}].$$

The conventional values for $K_{\mathrm{J}-90}$ and $R_{\mathrm{K}-90}$ fixed the size of a new set of *conventional electrical units* A_{90}, V_{90}, Ω_{90}, W_{90}, H_{90}, F_{90} and so on[4].

[4]The conventional units are written in italic type (e.g., A_{90}) in recognition of the fact that they are physical quantities, and the 90 subscript recognizes their introduction in 1990.

9 Conventional units drift apart from SI units

In the period 1990–2019 *all* electrical measurements and calibrations were performed in conventional units. This fact was mostly left unrecognized in measurement outcomes. In 1990 the only consequence was the expected one, an improvement of the measurement uncertainty thanks to the high reproducibility of the quantum standards.

The 1990 conventional units introduced, a century later, the very same problem of the 1893 practical international units [9]. Soon, new determinations of the Josephson and von Klitzing constant measured in terms of the SI units were published, and the 1989 values given above were updated, not only with improved accuracies, but also to different values. In 2014 the knowledge of the constants was the following [10]:

$$K_{\mathrm{J}} = 483\,597.8525(30)\,\mathrm{GHz\,V^{-1}} \qquad [6.1 \times 10^{-9}]$$
$$R_{\mathrm{K}} = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}],$$

from which it follows that

$$V_{90} = 1 + 9.8(6) \times 10^{-8}\,\mathrm{V}$$
$$\mathit{\Omega}_{90} = 1 - 1.764(2) \times 10^{-8}\,\Omega.$$

The shift of the constants' SI values from the conventional values makes the size of the conventional units different from that of the SI units. In particular, the size of the electromagnetic unit of power W_{90} is different from the size of the mechanical unit of power W. The overall resulting system is, as the 1893 one was, noncoherent. History repeats itself.

10 The revised SI

The existence of a parallel system of conventional electrical units, drifting from the corresponding SI ones, was a major problem of the SI[5]. The need of a profound revision of the SI was recognized at the turn of the century. The revision was approved by the 26th General Conference of Weights and Measures in November 2018 and implemented on 20 May 2019, the *implementation day*. The present SI is based on a set of seven constants with exactly specified numerical values.

[5]The main problem of the SI was the suspected instability of the international prototype kilogram, which defines the unit of mass.

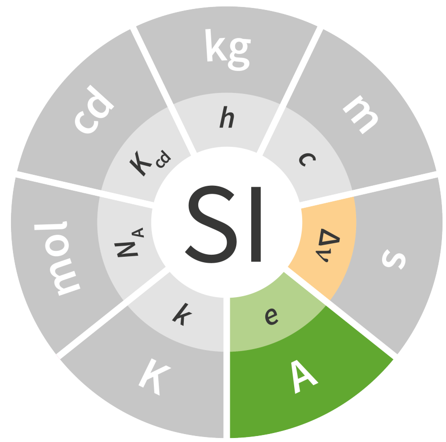

Figure 1: A pictorial representation of the SI, highlighting the base unit ampere (A) and the fundamental constants that enter the definition, the elementary charge e and the hyperfine transition frequency of the caesium atom $\Delta\nu$.

The definition of the ampere (Fig. 1) is

> The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\,176\,634 \times 10^{-19}$ expressed in the unit C, which is equal to A s, where the second is defined in terms of $\Delta\nu_{\mathrm{Cs}}$ [where $\Delta\nu_{\mathrm{Cs}}$ is the hyperfine transition frequency of the caesium atom].

In the definition of the kilogram, the SI fixes also the value of the Planck constant to be $h = 6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s}$. The values of e and h fixed by the revised SI correspond to the best last determination of the same constants in the previous SI.

Josephson and von Klitzing constants have fixed values with no uncertainty. The quantum electrical experiments are *realisations* of the electromagnetic

SI units.

No significant changes between the size of the SI units realized from the past and the revised SI definitions occur[6]. Measurements performed in the period 1990-2019, however, were expressed in the conventional 1990 units. Hence, a small step in the values of maintained electrical standards (calibrated in terms of the quantum standards both before and after the revision) occurs after the implementation day.

11 Conclusions

Measurement units must adapt to the scientific and technical discoveries and challenges and must therefore evolve with time. Recognizing this, the 2019 SI does not refer to any specific physical effects; hence, *any* experiment that directly links a quantity value to one or more of the fixed constants through known physical laws, with an uncertainty sufficient for the measurement purpose, can be considered a realisation of the SI unit for that quantity. The physics of quantum electromagnetic phenomena in solid-state systems is evolving quickly and we expect more to come in the near future.

The ideas of Giorgi of what constitutes a *good* system of units have aged well and remain at the basis of the 2019 revision of the SI. The kilogram can be and *is* now realised with the Kibble balance [11], which in the revised SI experimentally implements the equivalence of the electromagnetic and mechanical units of power and energy. I suspect that Giorgi would be amused by this hierarchy reversal, of mechanical units "made" from electrical ones.

References

[1] G. Giorgi, "Unità razionali di elettromagnetismo," in *Atti Associazione Elettrotecnica Italiana*, vol. V, 1901, pp. 412–418, comunicazione al Congresso di Elettrotecnica, Roma, 13 ottobre 1901.

[2] ——, "Rational units of electromagnetism," in *Proc. Phys. Soc.*, May 1902, communication to the Physical Society, London, 22 May 1902.

[3] W. Bich, "The third-millennium International System of Units," *Riv. Nuovo Cim.*, vol. 42, pp. 49–102, 2019.

[4] Bureau International des Poids et Mesures, "The International System of Units," May 20 2019, 9th SI Brochure. [Online]. Available: www.bipm.org

[5] F. Frezza, S. Maddio, G. Pelosi, and S. Selleri, "The life and work of Giovanni Giorgi: The rationalization of the International System of units [Historical corner]," *IEEE Antennas and Propagation Magazine*, vol. 57, no. 6, pp. 152–165, 2015.

[6] I. A. Robinson and S. Schlamminger, "The watt or Kibble balance: a technique for implementing the new SI definition of the unit of mass," *Metrologia*, vol. 53, pp. A46–A74, 2016.

[7] E. R. Cohen and B. N. Taylor, "The 1986 CODATA recommended values of the fundamental physical constants," *J. Phys. Chem. Ref. Data*, vol. 17, no. 4, pp. 1795–1803, 1988.

[8] B. N. Taylor and T. J. Witt, "New international electrical reference standards based on the Josephson and quantum Hall effects," *Metrologia*, vol. 26, no. 1, p. 47, 1989.

[9] L. Callegaro, "A quantum ampere," *tm-Technisches Messen*, vol. 87, no. 4, pp. 258–265, 2020.

[10] P. J. Mohr, D. B. Newell, and B. N. Taylor, "CODATA recommended values of the fundamental physical constants: 2014," *Rev. Mod. Phys.*, vol. 88, p. 035009, Sep 2016.

[11] S. Schlamminger and D. Haddad, "The Kibble balance and the kilogram," *Compt. Rend. Phys.*, vol. 20, no. 1, pp. 55–63, 2019.

[6] The differences are at the level of parts in 10^9.