

*I. Mihai*

**Low-frequency noise analyses in measurements  
of high standard resistance bridges**

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I.N.R.I.M. TECHNICAL REPORT

## Abstract

At the National Institute of Metrological Research (INRiM), the first evaluation of an exploratory model in the low-frequency noise in measurements of high-standard resistance commercial bridges and the low limits and high limits has been made using restrictive metrological triangulation rules. The study was performed by observing the ratio measurements of a commercial bridge for the standard resistors in the range from  $10\text{ T}\Omega$  to  $1\text{ P}\Omega$ , to calculate the type A uncertainty.

**Keywords:** High resistance measurements, equivalent ratio model, measurement uncertainty, self-calibration process, measurement noise, ISO 17025 and the risk assessment.

## Sommario

Presso l'Istituto Nazionale di Ricerca Metrologica (INRiM) è stata effettuata una prima valutazione esplorativa di un modello per il rumore a bassa frequenza delle misure con alti valori di resistenza resistori per un ponte commerciale a doppia sorgente e i valori inferiore e superiore per le regole di triangolazione metrologica restrittive. Lo studio è stato condotto osservando le misure di rapporto di resistenze campione nel campo da  $10\text{ T}\Omega$  a  $1\text{ P}\Omega$ , per il calcolo dell'incertezza tipo A.

**Keywords:** Misure di resistori campioni di alto valore, modello equivalente di rapporto di resistenze, incertezza di misura, test di compatibilità, misure affette da rumore, ISO 17025 e la valutazione del rischio.

## Opinions and comments from the Experts on this technical report

**Duane Brown**, Senior Member IEEE  
CEO, Measurements International, Canada

*I have been asked to write an opinion on a report titled “Low-Frequency Noise Analyses in Measurements of High-Standard Resistance Bridges” by Iulian Mihai, I.N.R.I.M. TECHNICAL REPORT, November 2023. The report is a continuation of Metrological triangulation rules in ratio measurements, May 2022, based on high-value resistance measurements from 10 TΩ to 100 TΩ using a Measurements International Model 6600A Dual Source Resistance Bridge.*

*This report aimed to evaluate the low-frequency noise in measurements of high-standard resistance commercial bridges with low limits and high limits being made using restrictive metrological triangulation rules. The study was performed by observing the ratio measurements for the standard resistors in the range from 10 TΩ to 1 PΩ to calculate the type A uncertainty.*

*The paper covers the general guidelines for technical papers covering Abstracts, Introduction, Hypothesis, Data sampling including Results and Discussions, Conclusion, and References. The paper is well written, featuring many diagrams and flow charts, and one can learn a lot about understanding the elements used in the resistors over the range of 10 TΩ to 1 PΩ, including two and three terminal elements.*

*The paper discusses the pros and cons of using elements such as wire wound, thin film, and thick film elements. Resistor tolerances based on heat, voltage, and frequency are discussed, along with stray inductance and capacitance. Detector specifications of the 6514 are discussed with the detector resolution for these experiments, which can be as low as 0.1 fA= 100 aA with a standard deviation of 28.9 aA.*

*The reason why large-value resistors have high noise is also explained. The paper is full of knowledge, data, and ideas.*

**Ladislau Matekovits**, Marie Curie Fellow, PhD

Associate Professor of Electromagnetic Fields  
Department of Electronics and Telecommunications, Politecnico di Torino, Italy

*The report entitled “Low frequency noise analysis in measurements of high standard resistance bridges” authored by Dr. Ing. Iulian Mihai from INRiM, Torino, discusses important aspects related to the noise in high precision resistors of  $T\Omega$  values. According for example to [30], a joint work between the Istituto Nazionale di Fisica Nucleare (INFN) and Laboratori di Gran Sasso Italy, when a resistor is biased with a constant current, fluctuation of voltage can be observed. Such unwanted variations follow a simple law; if properly handled, it allows extraction of information related to the noise of such resistors. The topic is of huge scientific and industrial interest, since the above-mentioned resistors are used during/in calibration phase. Hence even a small error in the nominal value can have a strong influence on the secondary set of resistors. Considering the application field of such high resistance components, e.g., ionization current measurements, their accurate characterization is even more critical.*

*The proposed model has a general validity. The expression in eq. 26 describing the LF noise variances has two independent terms: one is inversely proportional to the frequency and incorporates a technological parameter. The second term corresponds to the Brown oscillation, and accordingly it depends on the temperature and bandwidth. Separating the intrinsic noise and fabrication process dependent influences, allows to concentrate the attention on the controllable part, i.e., the process dependent one.*

*The LF noise of the thick film resistor, which is proportional to the power of its bias voltage, may yield a contribution to the resistor total noise much larger than thermal noise; the  $1/f$  noise is an (uncorrelated) excess noise. It has been remarked that the application of bias ( $V_x$  and  $V_s$ ) does not generate LF noise, but simply enhances an already present fluctuation mechanism. Surface and bulk atomic motions were identified as fundamental source (microscopic source) of  $1/f$  noise.*

*Apart of the scientific importance of the topic, I personally appreciated the clear explanations, supported by equations and high-quality graphs. The reading of the almost 100-page report is very pleasant.*

**Marcus Vinícius Viegas Pinto**, M.Sc. and PhD Student

Researcher-Technologist in Metrology and Quality at Inmetro, Brazil

*The technical report is very well written and presents a series of evaluations and analyzes of the noise present when measuring high-value resistors using the modified Wheatstone bridge, which are very useful to better understand all aspects involved in this type of measurement. It also uses a tool that helps guarantee the validity of the results through the triangulation technique, which proves to be quite applicable in this context.*

## SUMMARY

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## 1. INTRODUCTION

The previous technical reports explored the metrological triangulation rules in ratio measurements using the technique of Allan variance and the power spectral density to improve the performance in high resistance measurement systems and to solve the systematic errors or functioning errors of the commercial bridge MI 6600A. In this technical report, a first evaluation of an exploratory model of the low frequency noise for a commercial dual source high resistance bridge has been made using analogous model available from literature and Monte Carlo simulations. The restrictive metrological triangulation rules, using the measurements from the previous work has been completed by adding low limit rules, coming from the low frequency noise analyses. The study was performed observing the ratio measurements of a commercial bridge for the standard resistors in the range from 10 T $\Omega$  to 1 P $\Omega$ . To perform this work, free software such as Stable32, Calibration Curves Computing – CCC Software and Monte Carlo simulation by the software LNE Uncertainty has been used as well as free of charge eBooks from Wikibook (see page 19). This work emphasis the opportunity for metrologist and students to use open-source software and free of charge eBooks to check the triangulation rules for their own high resistance measurement systems. The predominantly aim of this work is to identify a method of measurement that is more efficient and more reliable, with respect to the traditional method, in order to successively carry out calibration services in terms of the ISO 17025 or, for example, measurements of bulk resistors, network resistors or the insulation materials of composite structures of the thermal shields in the range from 100 k $\Omega$  to 1 P $\Omega$ .

The author wishes to express his profound thanks to Mr. **Duane Brown** from Measurements International, Canada for his help with the 6600A bridge's operation, to Prof. **Ladislau Matekovits** for the opportunity to collaborate on topics as low frequency noise of scientific interest and Mr. **Marcus Vinícius Viegas Pinto**, researcher from Inmetro, Brazil for his interest in application of measurements of high ohmic standard resistors and applications scientific and industrial field. The Appendix **B. Analysis of acceptance limits using Monte Carlo simulation** has been edited by Mr. Pinto, based on his PhD program.

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