

RealMass Calibration Software Release 1.0

User Manual

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Andrea Malengo and Davide Torchio

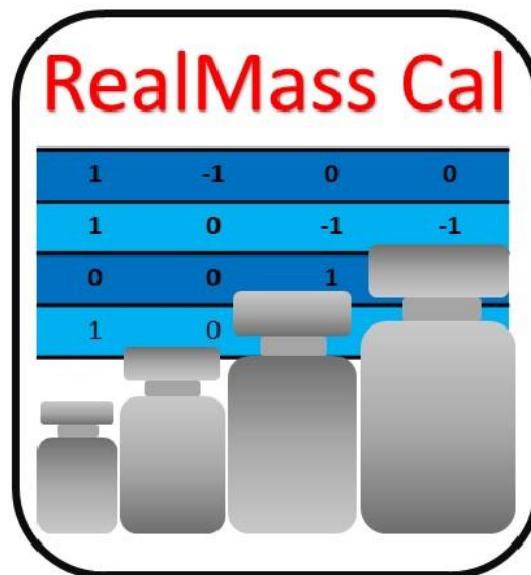


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1 Introduction

This document is the User Manual for the software “RealMass Calibration” (Release 1.0). This software has been developed under Matlab environment by the Istituto Nazionale di Ricerca Metrologica (INRiM). The purpose of the software is to be a tool for mass standards calibration.

This software was the main component of the activity A3.2.6 in Work Package 3 Task 3.2 “Development of advanced mathematical and statistical tools for estimating results and the associated uncertainties” of the EMPIR project 19RPT02 RealMass “Improvement of the realisation of the mass scale” [1].

The software has been realised for mass standards calibration activities, in particular for the realisation of the mass scale. However, its versatility and user-friendly graphical interface make it appropriate also for simple one-to-one calibrations. The methodology applied is that of the design matrix, which provides the scheme of comparisons between the different mass standards involved in the calibration. The process is based on the determination of the mass difference between weights, or groups of weights, of equal nominal value. To estimate the unknown values, the software uses the two least squares methods most widely used in mass metrology: the weighted least squares by Lagrange multipliers and the Gauss-Markov approach.

A brief description of the software is provided in ref. [2]. In this User Manual the use of the software is explained in more detail.

This manual and the User Licence Agreement can be open from the menu toolbar.

2 Licence Agreement

Release 1.0 of the software is provided with a Software User Licence Agreement, which is fully reported in Subsection 2.1, and the use of the software is subject to the terms laid out in that agreement. By installing and running the software, the User accepts the terms of the agreement.

To run the software, the User must first install the MATLAB [3] Component Runtime (MCR) libraries.

The MCR installer is not included in the software distribution, but it may be obtained by downloading from <http://www.mathworks.com/products/compiler/mcr/index.html>. Information on how to obtain the MCR installer file is given in the file README.txt included in the software distribution. The User must accept the terms of the MCR Library License as part of the installation of the MCR libraries.

2.1 RealMass Calibration, Software User Licence Agreement

This User License Agreement of the software “RealMass Calibration” is made and is effective on the first day the User installs the Software, by and between the Istituto Nazionale di Ricerca Metrologica (hereinafter referred to as “INRiM”), the Italian National Metrological Institute (NMI), and you, the “User”, either acting on behalf of your organisation or yourself.

It is hereby agreed that the User will be permitted to use INRiM’s RealMass Calibration Software (hereinafter referred to as the “Software”) free of charge for the express purpose of his own use in accordance with the following terms and conditions of this Agreement:

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The User also understands that they will NOT make use of (or be permitted to gain access to) INRiM RealMass Calibration Software if they are unable to accept and/or comply to the terms of this Licence Agreement unless a written agreement is obtained from the INRiM.

*INRiM, Istituto Nazionale di Ricerca Metrologica
Strada delle Cacce 91, 10135 Torino (Italy)
Telephone: +39 011 3919.1 (switchboard)
Fax: +39 011 346384
Email: INRiM@INRiM.it
VAT Number: IT09261710017*

3 Installing the software

The software takes the form of an application called `RealMass_Cal.exe`, stored, together with the following files:

- `RealMass Calibration Software_Licence_Agreement.pdf;`
- `User_Manual.pdf;`
- `ini.txt`
- `Readme.txt`
- `Splash.png`

And the Excel files with the examples:

- `DB example.xlsx`
- `Example C1.xlsx`

- Example C2.xlsx
- Example C3.xlsx

The software is intended to run on a personal computer running with Microsoft Windows operating system (64 bit) and Microsoft Excel.

The application software can be run as follows:

- Verify that version 9.11 (R2021b) of the MATLAB Runtime is installed. If not, read the “readme.txt” file for the installation.
- Copy all software files and the file “DB example.xlsx” to a folder in which the User has write permission for the files.
- Double-click on the executable file “RealMass_Cal.exe” in Windows Explorer, or create the shortcut to the file on the desktop. A window opens and the main graphical user interface (GUI) is displayed (it may take several seconds for the GUI to be displayed);

In case the software does not start, check that:

- The “ini.txt” file is contained in the same folder as the executable file
- The content of the “ini.txt” file indicates the Excel database file in use, on first installation it will contain the text “DB example.xlsx”
- Check the existence of the Excel database file indicated in the “ini.txt” file

The “ini.txt” file is updated by the software, however in the event of mistakes by the User, such as removing the database file from the folder, the “ini.txt” file can be restored by the User by entering in the file the name of the database in use (complete with the path).

4 Using the software

4.1 General

The GUI, shown in Fig. 1, is divided into several parts that allow the User to perform the following operations:

SELECT INPUT DATABASE: the User selects a Microsoft Excel workbook. The file contains data regarding the weights, balances and calibration parameters.

INPUT CALIBRATION PARAMETERS: these parameters are used for the data processing and can be set by the User.

SELECT INPUT FILE Weighing Plan: the User selects a Microsoft Excel workbook. The file contains data regarding the weighing plan, that is the list of the weights involved in the calibration.

SELECT INPUT FILE Weighing Design: the User selects a Microsoft Excel workbook. The file contains the design matrix.

SELECT INPUT FILE Measurements: the User selects a Microsoft Excel workbook. The file contains the measurements. A pop-up menu allows to select the grouping method of the weights.

Weighing Design Table: the table shows the “ID” of the weights and the design matrix.

Results Table: the table shows the “ID” of the weights and the results.

SELECT STATISTICAL MODEL: the User selects the statistical model to be used.

PROCESSING DATA: the User presses the `ELABORATION` button for the data elaboration. Two test cells show the standard deviation of the fit and the value of the test residuals. With the `Recalculation` button it is possible to recalculate the estimates taking into account the values of the residuals. Two pop-up menu allow to select how to consider the residuals in the additional uncertainty contribution.

SELECT OUTPUT FILE: the User saves the results to an output data file. By the `RECORD` button, the User can record the results in a file with the historical calibrations. A text cell shows the User name.

Each function is described in detail in the following subsections.

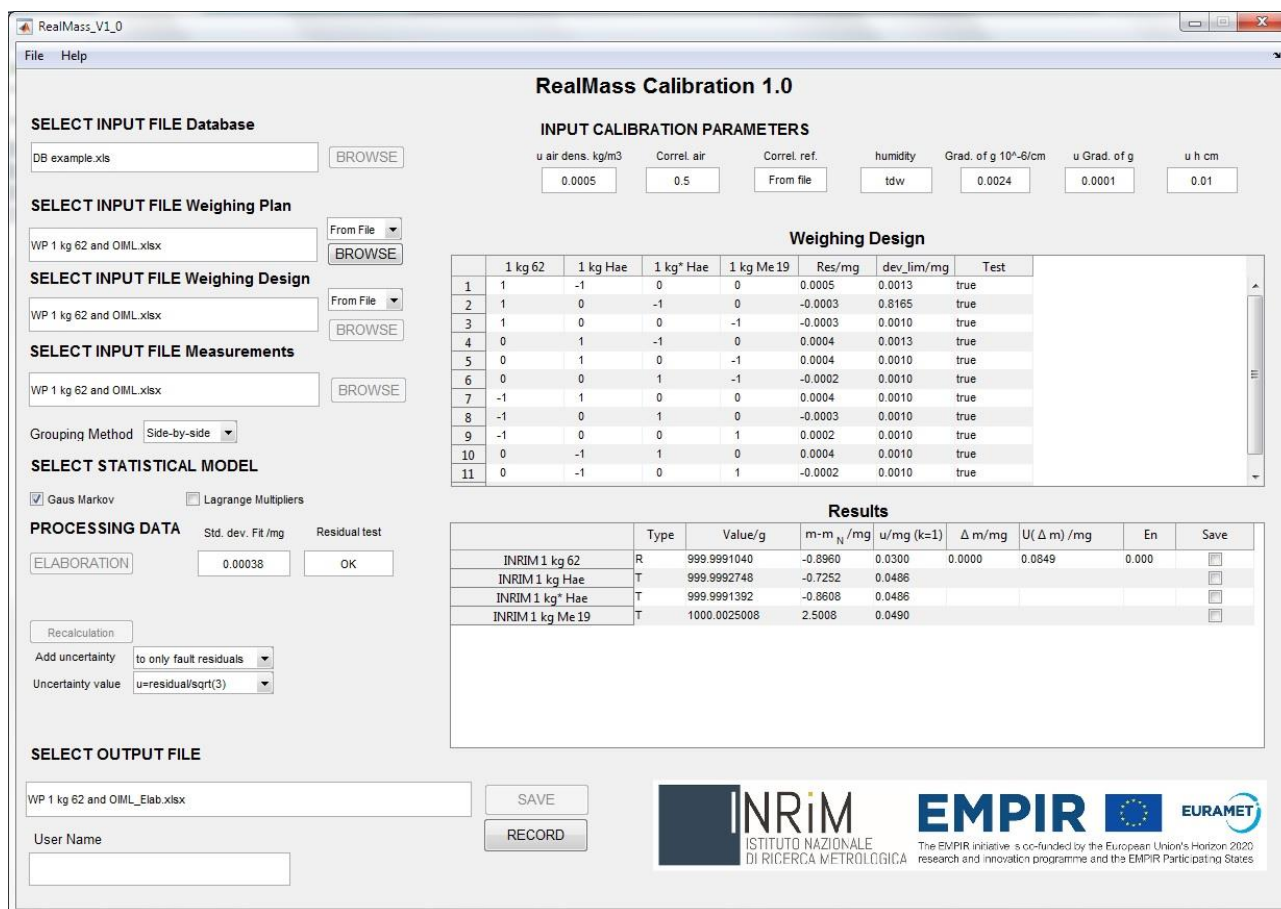


Figure 1. The GUI of the RealMass Calibration software

4.2 SELECT INPUT FILE Database

This function is used to define the characteristics of all weights that will be used in calibrations, both the reference, check and under test mass standards. There is also the possibility for the User to include the characteristics of the balances and some calibration parameters used for the data processing.

The text cell shows the name of the database in use. When the software is started, the last file used in the previous use is loaded. The name of the file in use, including the path, are stored in the “ini.txt” file, located in the directory where the RealMass Calibration software executable file is located.

Pressing the `BROWSE` button allows the User to select a Microsoft Excel workbook, i.e., a .xls or a .xlsx file.

On pressing the `BROWSE` button, the software opens a browsing window. The default directory proposed by the software is the directory where the main MATLAB script is located, but the User can browse other directories by the usual commands of the browsing window.

When a new file is opened, the text cell and the “ini.txt” file are updated with the name of the new database.

The loading of a new database can only be performed at the start-up of the software. After any other command of the software has been executed, it is no longer possible to modify the database from the GUI, in this case, if it is necessary to load a new database, it is necessary to restart the software.

The database file contains all information concerning the weights, the balances, and information on some parameters required for the calibration.

The input data file is required to have three worksheets titled “Data”, “Balances” and “Info”, filled in as described in the following.

The data entered in these worksheets (“Data”, “Balances”, “Info”) are checked by the software. In the presence of mistakes, such as: non-numerical data, values outside the measurement range, correlations with negative values or greater than 1, etc., the software will alert the User of the error. A detailed list of errors is shown in Appendix B.

On first start-up, the database file loaded is the example file “DB example.xlsx”. This file contains the data for the examples in this user manual. This file can be used by the User as a template.

It will be the task of the User to create a new database with the data of the mass standards and balances, and to load this new file. The databases can be copied in any folder of the computer.

Worksheet “Data”

The data in this worksheet relate to the characteristics of weights. The description of the worksheet is given in Table 1 and an example is shown in Fig. 2.

Header names in the worksheet must always be present even if the data fields are not used.

The User has to provide the density or the volume of each weight. If both values are provided, the volume value will be used in the calculation.

Column 11 is used to indicate the correlation between volume and mass for the reference standards. This information is optional, more information will be provided in section 5.2 and Appendix A. The instability uncertainty, column 14, is only taken into account for the uncertainty evaluation of the reference standards, the uncertainty of the results of the standards under calibration does not take into account the instability uncertainty.

The worksheet has to contain information for all the mass standards involved in the calibrations.

In the case of standards whose value is unknown, in column 3, the nominal value shall be entered, and in column 4 any value.

The weights included in the weighing plan are selected by the function *SELECT INPUT FILE Weighing Plan*. Weights are identified according to “Set of weights” name and “ID” name, the maximum number of characters of the string consisting of the names Set + ID is 65. Weights can have the same “ID” if the Set name is different.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	Set of Weight ID	Nom [g]	Value [g]	u (k=1) [g]	Volume [cm	u(k=1) [cm	Density [kg/m	u(k=1) [kg/m	HCoG [cm	h [cm]	Alfa x10 ⁻⁶	u(k=1) [1/°C]	u inst (k=1)	r(m,V)	Ref. Cal.	Cal. Date	
2	INRIM	1 kg 62	1000	999,999104	0,00003	46,435	0,02			1,95	3,9	25,982	2			xxxx	10/10/2022
3	INRIM	1 kg H	1000	999,999204	0,0000147	124,23063	0,0003			3,5	7,5	45	2,5				
4	INRIM	1 kg* H	1000	999,999076	0,0000147	124,23191	0,00048			3,5	7,5	45	2,5	1,5E-06		inrim 12	11/10/2022
5	INRIM	1 kg M 19	1000	1000	1	126,1415	0,0012	8000	8	3,5	7,5	45	2,5				
6	AB	500 g	500	500,000100	0,00005			7950	2								
7	AB	200 g	200	200,000000	200,00003	25,012	0,0015									xxx	12/02/2023
8																	

Figure 2. Example of the “Data” worksheet

Column	Header	Description	Unit	Information	Symbol
1 (A)	Set of the Weights	Set identification name, which includes a set of mass standards	-	Required	-
2 (B)	ID	Identification name of the weight	-	Required	-
3 (C)	Nom [g]	Mass nominal value	g	Required	m_N
4 (D)	Value [g]	Mass value (true mass)	g	Required	m
5 (E)	u (k=1) [g]	Standard uncertainty of the mass value	g	Required	$u(m)$
6 (F)	Volume [cm ³]	Volume at 20 °C	cm ³	Required if density is not provided	V_{20}
7 (G)	u(k=1) [cm ³]	Standard uncertainty of volume	cm ³	Required if volume is provided	$u(V_{20})$
8 (H)	Density [kg/m ³]	Density at 20 °C	kg/m ³	Required if volume is not provided	ρ_{20}
9 (I)	u(k=1) [kg/m ³]	Standard uncertainty of density	kg/m ³	Required if density is provided	$u(\rho_{20})$
10 (J)	HCoG [cm]	Centre of gravity	cm	Optional. If not provided, the value is 0 cm	h_{CoG}
11 (K)	h [cm]	Height	cm	Optional. If not provided, the value is 0 cm	h
12 (L)	Alfa x10 ⁻⁶ [1/°C]	Coefficient of expansion	1/°C	Required	α
13 (M)	u(k=1) [1/°C]	Standard uncertainty of the coefficient of expansion	1/°C	Required	$u(\alpha)$
14 (N)	u inst (k=1) [g]	Uncertainty of instability	g	Optional. If not provided, the value is 0 g	u_{inst}
15 (O)	r(m,V)	Correlation between Volume and Mass values	-	Optional. If not provided, the value is 0	$r(m, V)$
16 (P)	Ref. Cal.	Traceability of calibration (document, file, etc)	-	Optional	-
17 (Q)	Cal. Date	Calibration Date	-	Optional	-

Table 1. Content of the "Data" worksheet

Worksheet "Balances"

The information in this worksheet allows the calculation of some weighing parameters, such as corrections to the balance readings and evaluation of the uncertainties due to the balance, depending on the indicated characteristics of the balance in use. However, the User can choose to indicate no balance at all, in which case it will be up to the User to correct the readings of the balances and evaluate the uncertainties. More information is given in Appendix A.

The description of the worksheet is given in Table 2 and an example is shown in Fig. 3.

Column	Header	Description	Unit	Information	Symbol
1 (A)	Model	Identification name of the balance	-	Required	-
2 (B)	Capacity [g]	Capacity	g	Required	-
3 (C)	Resolution [mg]	Resolution	mg	Required	res
4 (D)	Adj mass [g]	Mass value of the adjustment mass standard	g	Required	m_{Adj}
5 (E)	u adj mass [mg]	Standard uncertainty of the adjustment mass standard	mg	Required	$u(m_{Adj})$
6 (F)	Dens Adj mass [kg/m ³]	Density of the adjustment mass standard	kg/m ³	Required	ρ_{Adj}
7 (G)	u ecc [mg]	Standard uncertainty contribution due to the eccentricity	mg	Required	u_{ecc}
8 (H)	u lin [mg]	Standard uncertainty contribution due to the linearity	mg	Required	u_{Lin}
9 (I)	Read Adj mass [g]	Reading of the balance with the adjustment mass standard	g	Required	Δm_{Adj}
10 (J)	pos	Difference between two positions, with the format x_y	-	Optional.	-
11 (K)	corr [mg]	Correction of the balance difference	mg	Optional.	Δm_E

Table 2. Content of the "Balances" worksheet

The headings of columns 1 to 9 must always be present even if no balances have been indicated. If a balance model is indicated, all column fields from 1 to 9 must be filled in.

From column 10 onwards there is the option to enter corrections for possible position errors for automatic balances with several weighing positions. If empty, no correction will be applied.

For example, if the balance is affected by an error when the difference is between the positions 2 and 3, the software allows this error to be corrected automatically. In column 10 enter 2_3 and in column 11 the correction to be applied to the difference (in mg).

If correction 2_3 is entered, automatically the software defines correction for 3_2 by reversing the sign of correction 2_3.

For example, if A is in position 2 and B is in position 3, and correction 2_3 is Δm_E (mg), be the measured mass difference A-B = $\Delta m_w'$, the software corrects the difference A-B as $\Delta m_w' + \Delta m_E$.

Add in the subsequent columns the corrections for other positions. Positions with null corrections are not required to be indicated.

The balances used for each weighing will be indicated in the worksheet “Meas”.

Balances are recognized according to the “Model” name.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Model	Capacity [kg]	Resolution [kg]	Adj mass [kg]	u adj mass [kg]	Dens Adj [kg/m³]	u ecc [mg]	u lin [mg]	Read Adj mass	pos	corr [mg]	pos	corr [mg]	
2	M1kg	1000	0,001	10	0,005	8000	0	0	10	1_2	0,0015	2_3	-0,0015	
3	CC1	1000	0,01	1	0,003	8000	0	0	1					
4	ABC	100	1	10	1	8000	0	0	10					
5														
6														
7														
8														

Figure 3. Example of the “Balances” worksheet

Worksheet “Info”

This worksheet contains information on some typical laboratory calibration parameters, which are normally common to all measurements. The description of the worksheet is given in Table 3 and an example is shown in Fig. 4.

Column	Header	Description	Unit	Information	Symbol
1 (A)	u air dens [kg/m3]	Standard uncertainty of the air density	kg/m ³	Required	$u(\rho_a)$
2 (B)	air dens correl	Air density correlation	-	Required	$u(\rho_{ai}, \rho_{aj})$
3 (C)	references correl	Reference standards correlation	-	Required	$u(m_{refi}, m_{refj})$
4 (D)	g gradient [10 ⁻⁶ /cm]	Gradient for g	×10 ⁻⁶ cm ⁻¹	Required	$\Delta g/g$
5 (E)	u (g gradient)	Standard uncertainty for gradient for g	×10 ⁻⁶ cm ⁻¹	Required	$u(\Delta g/g)$
8 (F)	u(h) [cm]	Standard uncertainty for centre of gravity and height of the weights	cm	Required	$u(h_{CoG})$
7 (G)	hr%/tdw	Indicates whether the humidity measurement is in terms of relative humidity “hr%” or dew point temperature “tdw”	hr% or °C	Required	-

Table 3. Content of the “Info” worksheet

	A	B	C	D	E	F	G	H
1	u air dens [kg/m3]	air dens correl	references correl	g gradient [10 ⁻⁶ /cm]	u (g gradient u(h) [cm]	hr%/tdw		
2	0,0001	0,5	0,95	0,0024	0,0001	0,01	tdw	
3								
4								
5								
6								
7								
8								

Figure 4. Example of the “Info” worksheet.

The values given for these parameters are indicated in the GUI in the “Input Calibration Parameters” section, apart from the “ur%/tdw” value, the other parameters can be modified by the User via the GUI. Changes to these data can take place before the data processing.

4.3 SELECT INPUT FILE Weighing Plan

This function is used to select the weights involved in the weighing scheme, including reference, check and under calibration mass standards. The selected weights define the column vector m for the parameters of the system of equations (see Appendix A).

The BROWSE button has two functions, depending on the pop-up menu setting:

- Pressing the BROWSE button and pop-up menu on “From File” allows the User to select a Microsoft Excel workbook, i.e., a .xls or a .xlsx file already prepared with the weighing plan.
- Pressing the BROWSE button and pop-up menu on “New” allows User to select the Weights from the Database by the GUI, for defining the weighing plan and save a file which can be also used in the future elaborations.

The file to upload contains the identification names of the weights used for the weighing plan.

It can be prepared by the User by editing an Excel file, or more simply by means the BROWSE button and pop-up menu on “New”.

The input data file should have one worksheet titled “WP” and filled in as described in Table 4. An example is given in Fig. 6. The template is given in the example files.

Column	Header	Description	Unit	Information	Symbol
1 (A)	Set	Set identification name	-	Required	
2 (B)	ID	Identification name of the weight	-	Required	
3 (C)	Nom.Value/g	Nominal value of the weights	g	Required	m_N

Table 4. Content of the "WP" worksheet.

On pressing the `BROWSE` button and pop-up menu on “From File”, the software opens a browsing window. The default directory proposed by the software is the directory where the main MATLAB script is located, but the User can browse other directories by the usual commands of the browsing Window.

On pressing the `BROWSE` button and pop-up menu on “New”, a window, containing three tables, is shown, from which the weights can be selected from the database in use, as an example see Fig. 5. With the two tables on the left it is possible to browse the database, in the first there is the list of the names of the “Set of Weights”, in the second table appears the list of the “ID” names and nominal value of the correspondent Set selected. The table on the right contains the selected weights. The selection is made by selecting with the mouse the required weight (highlighted in blue) and then pressing the `ADD` button. The `REMOVE LAST` button can be used to delete the last line in the list. If a weight is selected twice an error message appears.

Once the list has been defined with all the weights, the User can save an Excel file with the `Save List` button.

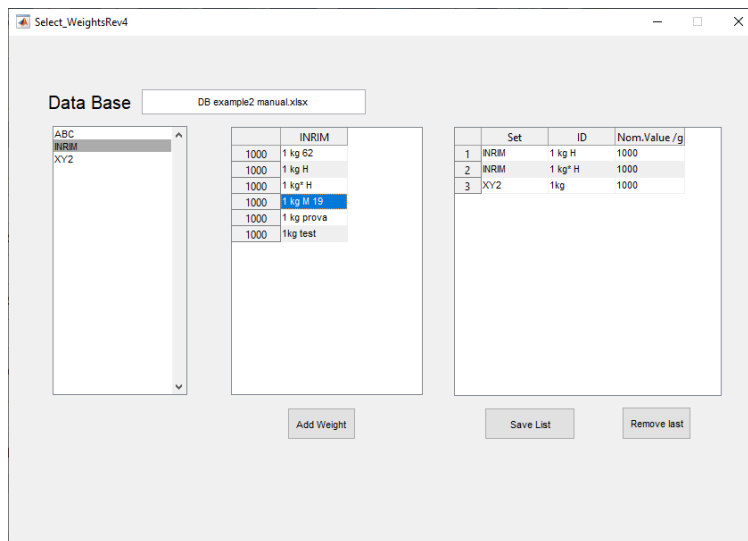


Figure 5. Selection of the weights by the GUI

	A	B	C	D	E
1	Set	ID	Nom.Value/g		
2	Nim	100 Nim	100		
3	Nim	100 Nim*	100		
4	H15	50 h*	50		
5	H15	50 h**	50		
6	Nim	20**	20		
7	Nim	20***	20		
8	Nim	10	10		
9	Nim	10*	10		
10					
11					
12					
13					
14					
15					
16					
17					
18					

Figure 6. Example of the “Info” worksheet

After setting up the weighing plan, the selected weights are shown in the “Results” table of the GUI. It is now possible to define which are the reference, under test (that is, in calibration) and check standards. This is done by means of the table “Results”. At the beginning the table consists of two columns, the first indicating the name of the weight, that is the composition “Set of weights” and “ID”, the second the “Type” of standard (Fig.7).

For the second, the “Type” is indicated as:

- R for reference
- T for under test
- C for check

Check standards are treated in the same way as under test standards, the difference is that at the end of the processing, the normalised error E_n is calculated by taking the previous values (in the database file) into account (see Appendix A). The consistency of the calculated value with the previous value can be used as a validation of the result obtained. For this reason, ensure that the mass values and uncertainties provided are correct, both for reference and check standards. The E_n value is also provided for the reference standards, this parameter is important when using multiple references (see example C1). The use of the check standard is useful to quickly control the change from the previous value.

By default, the first one in the list of the weighing plan is the reference and the others are under test, however, the User has the option of changing the settings. It is possible to change the “Type” of each standard by double-clicking on the second column of the “Results” table and editing the “Type”: R, T, C, see Fig.7.

To facilitate this function, the User can add this information by entering the letters R, C or T in the fourth column of the “WP” worksheet (Fig. 8). This function is useful for particularly complex weighing schemes, where, for example, several reference and check standards are involved and these calibrations are repeated periodically.

Results					
	Type	Value/g	$m - m_N$ /mg	u /mg (k=1)	$\Delta m/n$
Nim 100 Nim	R				
Nim 100 Nim*	T				
H15 50 h*	T				
H15 50 h**	T				
Nim 20**	T				
Nim 20***	C				
Nim 10	T				
Nim 10*	T				

Figure 7. Definition of standard type by the GUI

	A	B	C	D	E
1	Set	ID	Nom.Value/g		
2	INRIM 2	1 kg E2	1000 R		
3	INRIM 2	1kg T	1000 T		
4	INRIM 2	500 g	500 R		
5	INRIM 2	500 g T	500 T		
6					
7					
8					

WP WD Meas +

Figure 8. Definition of standard type by the “WP” worksheet

With the Gauss-Markov method, any number of reference standards can be used, with the Lagrange multipliers method only one reference can be used.

When using several reference standards, it is necessary to evaluate their correlations, otherwise there is a risk of underestimating the uncertainties. More information on the evaluation of the correlations are given in section 5. Of course, there is no concern about correlation when using the Lagrange multipliers method, implying only one reference.

The User can provide the correlation coefficient with two options:

- 1) A value, the same for all weights, can be defined in the “Info” worksheet. This value is shown in the cell “Correl. Ref.” on the GUI, the correlation can be changed by editing the value in the text cell.
- 2) A value can be defined for each pair of standards, by means of an Excel worksheet, edited by the User. In this case, the file containing the weighing plan, that is with the worksheet “WP”, has to also contain a worksheet titled “Corr”. The correlations are indicated using an upper triangular matrix. In the case of a null correlation, the corresponding cell can be left empty.

To facilitate the construction of the matrix, it is possible to indicate the “ID” names of the weights in the first row and first column. The dimension of this matrix (excluding the first column and the first row) must be equal to the number of weights of the weighing plan, and the diagonal of '1' must be complete even if there is no correlation.

As an example, assuming that the vector of the weights is A, B, C and D, and the correlation between A and B is 0,5; between A and C is 0,4; and between C and D is 0,8, the Excel worksheet is shown in Fig. 9.

	A	B	C	D	E	F
1		A	B	C	D	
2	A	1	0,5	0,4		
3	B		1			
4	C			1	0,8	
5	D				1	
6						
7						
8						
9						

Figure 9. Example of “Corr” worksheet.

If the worksheet “Corr” is provided, in the text cell “Correl. Ref.” on the GUI the indication “From file” is shown.

Option 1 is applicable when only two reference standards are used, or in the case of more than two reference standards if they have similar correlations; this is normally the case when the standards have the same nominal value and their uncertainties are similar. In all other cases, it is recommended to use option 2 (see example C2).

For the sake of simplicity, the file containing the “WP” worksheet can contain all the information required for the calculation. In this case, it is necessary to add to the file containing the worksheet named "WP" (and “Corr”, if necessary) the other worksheets named:

- “WD” for the weighing design
- “Pos” (if necessary) for the positioning of the weights
- “Meas” for the measurements

The contents of these worksheets are described in the following sections. In this way, the file is complete with all the data, which are loaded all at once.

4.4 SELECT INPUT FILE Weighing Design

This function is used to define the design matrix X_w (see Appendix A), that is the weighing scheme which defines the differences between weights or groups of weights.

The design matrix is a matrix of 0, 1 and -1, which multiplied by the column vector of the parameters, defined by the selected weights, determines the weighing equations.

The BROWSE button has two functions, depending on the pop-up menu setting:

- Pressing the BROWSE button with the pop-up menu on “From File” allows the User to select a Microsoft Excel workbook, i.e., a .xls or a .xlsx file.

- Pressing the `BROWSE` button with the pop-up menu on “New” allows the User to define the design matrix by the GUI.

The input data file to upload should have one worksheet titled “WD” and filled in as described in the following:

- The excel file contains the design matrix, where each row multiplied by the vector of the parameters, corresponds to a weighing difference. The number of columns corresponds to the number of weights; the number of rows corresponds to the number of weighing equations (see Fig. 10).

	A	B	C	D	E	F	G	H	I
1	1	-1	0	0	0	0	0	0	0
2	1	0	-1	-1	0	0	0	0	0
3	0	1	-1	-1	0	0	0	0	0
4	0	0	1	-1	0	0	0	0	0
5	0	0	1	0	-1	-1	-1	0	0
6	0	0	0	1	-1	-1	0	0	-1
7	0	0	0	0	1	-1	0	0	0
8	0	0	0	0	1	0	-1	0	-1
9	0	0	0	0	0	1	-1	0	-1
10	0	0	0	0	0	0	1	0	-1
11									

Figure 10. Example of “WD” worksheet.

When loading the matrix, the software automatically checks that each weighing equation is meaningful, that is, that the nominal values of the weights, or combinations of weights to be compared are equal, otherwise an error message is shown. With this software it is not possible to define equations as sums of weights.

It is also checked that the number of linearly independent equations is sufficient to find the solution with only one reference standard, otherwise an error message is shown.

Even with this error message, the software will perform the calculation, but in this case the User is advised that more than one reference standard must be defined.

Pressing the `BROWSE` button with the pop-up menu on “New”, the User can set the design matrix using the GUI. This is done by means of a window containing two tables. The table on the left “Selected Weights” shows the list of weights that have been selected, the table on the right “Design Matrix” shows the design matrix under construction (see Fig. 11).

The equation for each row of the matrix is written by the User by editing the values 1, -1, or 0 in the third column “Equ” of the table “Selected Weights”.

By pressing the `Add` button the equation is included in the design matrix. Each row added must provide a weighing equation with the sum equal to zero otherwise an error message is shown. The `Remove Last` button can be used to delete the last line in the list.

The matrix is saved in an Excel file with the `Save` button.

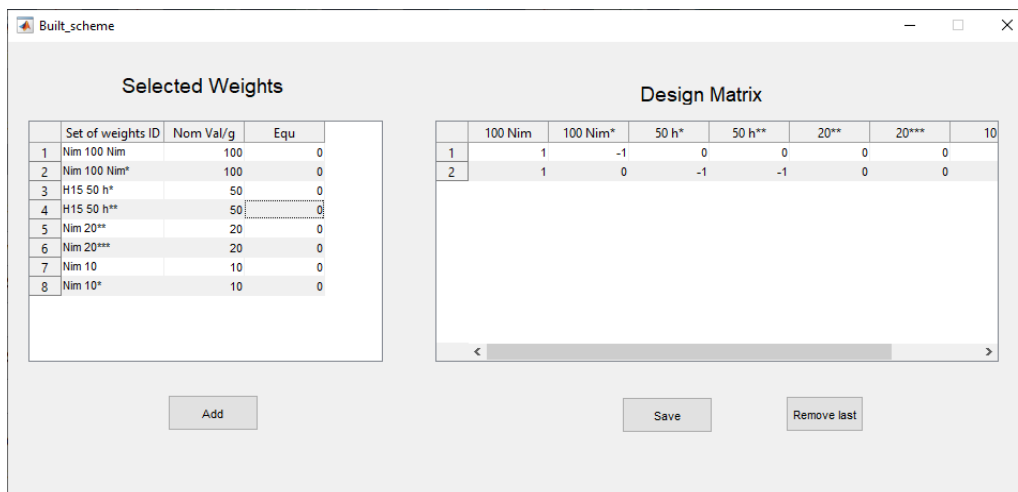


Figure 11. Implementation of the design matrix by the GUI.

Once the file is loaded, the design matrix together with the “ID” names of the weights are shown in the table “Weighing Design” on the GUI.

The difference between groups of weights is necessary in the case of calibration by subdivision or multiplication. From the design matrix, the groups of weights to be composed to perform the difference are therefore defined. Group A will be defined by all '1's and group B by all '-1's

The way in which the weights are grouped (stacked or side-by-side) has to be specified by the User, so that the software calculates the centre of gravity of each group of weights. From the difference of the centre of gravity between group A and B, the mass correction due to the gradient of the gravity is calculated. The mass correction for the difference of height is of the order of $3 \mu\text{g}$ per 1 kg per 1 cm (for $\Delta g/g = 0,003 \times 10^{-6} \text{ 1/cm}$), therefore this correction is negligible when comparing standards of similar height. Normally this correction also is negligible for weight groups with a total mass of less than 50 g.

If the User considers that the measurements should not be corrected for the effect of the different height, the User should enter the value zero (or leave the cell empty) for both the height of the weight and the height of the centre of gravity.

In order to define the grouping method, the User has the option to specify three possibilities:

- Side-by-side
- Stacked
- Custom

The “Side-by-side” method is when all the weights are side-by-side on the pan, for all the weighings.

The “Stacked” method is when all the weights of the groups are stacked, for all the weighing. This method is normally used when using disc-shaped weights.

These two methods can be selected from the GUI. In the case of stacked weights, the rule is that the weights with smaller diameters are on the top of those with larger diameters. The software estimates the approximate diameter of each weight from the volume and the center of gravity.

The “Custom” method must be used when the assembly consists of both side-by-side weights and stacked weights, or compositions of grouping methods. In this case, the User has to edit an Excel worksheet with a matrix that describes how the weights are positioned.

This worksheet called “Pos” will be included in the file that defines the design matrix (worksheet “WD”). This matrix should have the same dimensions of the design matrix.

In the matrix, for each weight involved in the weighing equation its position on the pan is indicated by a letter. The letter "z" (lowercase) indicates that the weight is located on the pan of the balance.

For any weight superposed to another weight, the letter (uppercase) corresponding to the column name of the Excel table in which the weight is located is used. The letter is identified by looking at the worksheet of the design matrix (worksheet “WD”).

If the weight is not part of the group, ‘0’ is entered.

There are controls on the implementation of the “Pos” matrix:

- At least one weight of each group must be placed directly on the pan of the balance (letter “z”)
- Letters must be uppercase and among those identifying the columns of the “WD” worksheet.
- The position of each weight may be on the plate (“z”) or on other weights involved in the weighing group.

Otherwise, an error message appears.

An example of weighing between two groups of weights is given in Fig. 12, the first row corresponding to the row for the worksheet “WD”, the second row corresponding to the row for the worksheet “Pos”.

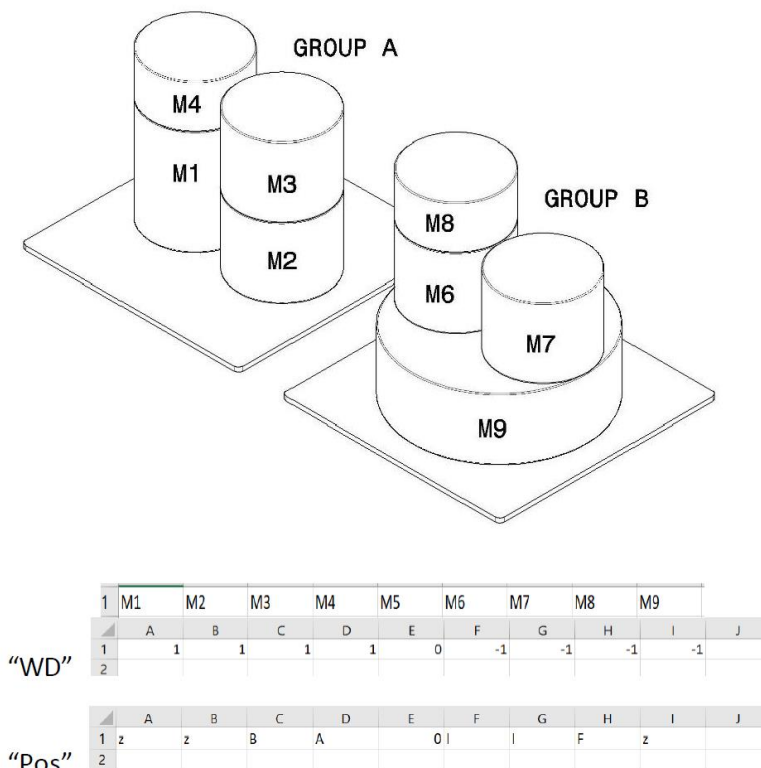


Figure 12. Example of grouping method.

If provided, this matrix must be complete with all information for all the weighing equations.

If the User defines the positions using the “Pos” worksheet, it is not possible to change the grouping method from the GUI. This function works for a maximum of 100 weights.

Depending on the chosen method, the software calculates the height of the center of gravity of the group of weights involved in the weighing (see Appendix A). An example is given in Example C1.

The gradient for the gravity and the associated uncertainty are set from the “Info” worksheet, the values are shown on the GUI. From the GUI the User can also change these values. From the gradient, the mass value and the difference of the center of gravity, the correction for the mass difference is evaluated (Appendix A).

4.5 SELECT INPUT FILE Measurements

The Excel file to load contains the measurements which are used for the determination of the vector of the weighing differences y_w (see Appendix A). Pressing the BROWSE button allows the User to select a Microsoft Excel workbook, i.e., a .xls or a .xlsx file. On pressing the BROWSE button, the software opens a browsing window. The input data file should have one worksheet titled “Meas” and filled in as described in Table 5, an example is shown in Fig. 13. The template is given in the examples file.

Column	Header	Description	Unit	Information	Symbol
1 (A)	Row	Row number, according to the row of the design matrix		Optional	-
2 (B)	Diff. Balance mg	Difference in apparent mass, determined by the balance	mg	Required	$\Delta m'_w$
3 (C)	Temp. °C	Temperature	°C	Required	-
4 (D)	hr %/tdw	Humidity in terms of hr % or tdw	hr % or °C	Required if the air density is not provided	-
5 (E)	Patm Pa	Pressure	Pa	Required if the air density is not provided	-
6 (F)	CO2 ppm	CO ₂	ppm	Required if the air density is not provided	-
7 (G)	rho air kg/m3	Air density	kg/m ³	Required if Humidity, Pressure and CO ₂ are not provided	ρ_a
8 (H)	u Diff mg	Standard uncertainty of the apparent mass difference	mg	Required	u_{diff}
9 (I)	Balance	Identification of the balance	-	Optional	-
10 (J)	Positions	Balance positions used	-	Optional	-
11 (K)	Date	Date of the measurements	-	Optional	-
12 (L)	Name	User Name	-	Optional	-
13 (M)	Note	Additional information	-	Optional	-

Table 5. Content of the "Meas" worksheet.

Headings in the worksheet must always be present even if the corresponding data fields are not used. If the air density in column 7 is provided, information in the columns 4, 5 and 6 is not required. If the air density in column 7 is not provided, information in the columns 4, 5 and 6 is required, as in this case the software evaluates the air density according to the CIPM 2007 equation [4]. If “Pressure”, “Humidity” and “CO₂”, and also air density values are provided, for the air density the software takes into account the air density value provided by the User.

Column 9 is to be used only if the User intend to automatically calculate the weighing uncertainties associated to the balance or to correct the readings by the sensitivity of the balance (see Appendix A).

Column 10 is to be edited if automatic balances are used and it is intended to apply corrections due to positions (if column 10 is used, column 9 must be used).

The last three columns indicate measurement information: “Date”, “Name” and “Note”.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Row	Diff. Balar	Temp. °C	hr%/tdw	Patm Pa	CO2 ppm	rho air	u Diff mg	Balance	Positions	Date	Name	Note
2	1	91,2814	19,06	8,51	98976,53	523	1,175381	0,0005	AT1	1_2			
3	2	91,4177	19,06	8,51	98976,55	523	1,175379	0,0005	ABC				
4	3	90,2999	19,06	8,51	98976,15	523	1,175374	0,0005					
5	4	0,1375	19,06	8,50	98976,13	523	1,175372	0,0005	AT1	3_2			
6	5	-0,9799	19,06	8,50	98976,35	523	1,175374	0,0005					
7	6	-1,1176	19,06	8,50	98976,35	523	1,175372	0,0005					
8	7	-91,2797	19,06	8,50	98976,35	523	1,17537	0,0005					
9	8	-91,4177	19,06	8,50	98976,48	523	1,175371	0,0005					
10	9	-90,2996	19,06	8,50	98976,43	523	1,175369	0,0005					
11	10	-0,1368	19,06	8,49	98976,13	523	1,175365	0,0005					
12	11	0,9801	19,06	8,49	98975,85	523	1,175363	0,0005					
13	12	1,1178	19,06	8,49	98976,08	523	1,175365	0,0005					
14													
15													
16													
17													
18													

Figure 13. Example of the “Meas” worksheet

4.6 INPUT CALIBRATION PARAMETERS

The text cells show the values of some calibration parameters, these values being automatically loaded from the “Info” worksheet of the database file. Before the data processing, the User has the option of modifying these parameters by the GUI.

In Table 6 the calibration parameters are described.

Header	Description	Unit	Symbol
u air dens. kg/m ³	Standard uncertainty of the air density	kg/m ³	$u(\rho_a)$
Correl. air	Coefficient of correlation for the air density measurements	-	$u(\rho_{ai}, \rho_{aj})$
Correl. ref	Coefficient of correlation for the reference standards	-	$u(m_{refi}, m_{refj})$
Humidity	States if humidity is measured in terms of hr% or temperature dew point tdw	-	-
Grad. of g 10 ⁻⁶ /cm	Gradient of g	10 ⁻⁶ /cm	$\Delta g/g$
u Grad. of g	Standard uncertainty of the gradient of g	10 ⁻⁶ /cm	$u(\Delta g/g)$
u h cm	Standard uncertainty of the height of the centre of gravity of the weights	cm	$u(h_{CoG})$

Table 6. Calibration parameters.

4.7 SELECT STATISTICAL MODEL

The User can choose between two estimators: Gauss-Markov or the Lagrange Multipliers method (see Appendix A).

The Lagrange Multipliers method can be used when only one standard is indicated as the reference. The reference standard can be in any position in the parameter vector (the vector containing the names of the weights of the “Results” table). For the Gauss-Markov method there are no limitations in the number of references. Ensure that the standards chosen as references have the correct mass and uncertainty values.

Although more than one reference standard could be considered with the Lagrange Multipliers method, this approach would only make sense when the relative uncertainties of the chosen reference standards with the same nominal value are equivalent. It would be inappropriate to apply the Lagrange Multipliers method using references with different uncertainties, as the resulting solution would not take into account these different uncertainties.

For this reason, the Lagrange Multipliers method is normally used by fixing only one reference standard, and this approach was also chosen for this software.

4.8 PROCESSING DATA

Before performing the calculation, the User should check that everything is correctly inserted, in particular the grouping method, the definition of reference standards and the associate values, the values of the calibration parameters and the statistical method for the elaboration.

On pressing the `ELABORATION` button, the calculations are performed, and the table “Results” shows the estimates of the parameters, the differences with respect to the nominal values, together with their associated standard uncertainties.

For the standards of type R and C, the difference between the new value and the initial value (the previous value from the database) and the normalised error E_n are also shown in the “Results” table. The calculation of E_n is performed considering the correlation between subsequent calibrations to be zero.

When using the Gauss-Markov method with more than one reference standard, the newly calculated values for the references may be different from those indicated in the worksheet “Data”. In this case, the consistency of the results must be checked (see example C1). If the results are not consistent, this means that there could be problems with the initial values of the reference standards.

Once the problem has been solved, the residuals (in mg) are shown in the “Weighing Design” table, in the column following the design matrix. In addition, the GUI shows the value of the standard deviation of the fit in the text cell “Std. dev. Fit/mg” (see Appendix A).

A test is also performed on the obtained residuals. The test consists of checking whether the residual for each equation is less than or equal to the calculated limit evaluated considering the balance resolution and the weighing uncertainty (see appendix A).

The column “Dev. Limit/mg” of the “Weighing Design” table shows this limit and on the last column of the table, “True” appears if the test is passed, otherwise “Fault”.

When the test is passed for all the residuals, the cell text “Residual Test” value of the GUI is “OK”.

Once the processing has been carried out, the User has the option of saving the results (see next section), or in the event that the results are not entirely satisfactory (and no errors in the measurements were found), the User has option to repeat the calculation (by the `Recalculation` button) taking into account the residuals obtained. The algorithm will increase the uncertainties as a function of the residuals. The use of this function does not guarantee that the results obtained will be consistent with the evaluated uncertainties.

Via two pop-up menus there is the possibility of establishing different methods to add the uncertainties according to the values of the residuals.

With the first pop-up menu for “Add uncertainty”, the options are:

- To all residuals: the additional uncertainty is considered for all the weighings.
- To only fault residuals: the additional uncertainty is considered only if the test of the residual is “Fault”.

With the second pop-up menu “Uncertainty value”, the calculation of the additional uncertainty u_{add} can be selected:

- $u = \text{Residual}/\text{sqrt}(3)$
- $u = \text{Residual}$

In case of recalculation of the parameters, in the last column of the “Weighing Design” table, the values of the additional uncertainties are shown.

After processing, the results can be save in an Excel file, or it is possible:

- to repeat the calculation by selecting a different estimator (Gauss-Markov or Lagrange)
- to change the “Type” for the standards (R,C,T)
- to change the calibration parameters, this last can be performed after deselecting or changing the estimator method
- to upload a new “Weighing Plan” file for a new calculation.

4.9 SELECT OUTPUT FILE

Once the elaboration is completed, the `SAVE` button becomes active, and, in the box itself, a name is automatically suggested for the output file: “FileName_Elab.xlsx”, where “FileName” is the name of the Excel input data file related to the “Meas” worksheet or to the “Weighing Plan” file (if this file contains all the worksheets).

On pressing the `SAVE` button, the software shows the browsing window “Select the Output data file”, which automatically opens the same directory in which the input Excel file is located. However, the file name and its location can be modified, placing the file in any desired directory. If such a file already exists in the selected directory, it is possible to overwrite it or to save it with a different name. The chosen file name is then displayed within the text cell of the GUI, and the `SAVE` button is disabled.

When the file is saved, the User has the option of entering his/her name. The name will be shown in the appropriate field in the GUI and will be recorded in the output file. Once the name is provided, it will also be used for subsequent elaborations, until the software is closed.

The output Excel file consists of seven worksheets:

1) “Results”

This worksheet contains:

- the version of the software
- the elaboration date
- the name of the input files
- the list of the mass standards and the results in terms of mass value
- the name of the User
- the grouping method
- the method used for the results estimation
- the standard deviation of the fit
- for the reference and the check standards, the deviation with respect the value in the input file database and the E_n value
- the covariance matrix of the results (in case of Gauss-Markov method)
- recalculation if performed

An example is given in Fig. 14.

	A	B	C	D	E	F	G	H	I	J
1	EMPIR 19RPT02 RealMass Software Release 1.0									
2	Istituto Nazionale di Ricerca Metrologica - INRIM, Italy									
3	Release Date: September 2023									
4	Developed in the framework of WP3, EMPIR Project RealMass									
5	Project financed by EURAMET									
6										
7	Date and time =06-Sep-2023 09:17:03									
8	User Name = Andrea									
9	File Data Base = C:\Users\malen\Documents\Real Mass\agosto 2023\DB example.xlsx									
10	File WP = C:\Users\malen\Documents\agosto 2023\WP 100 g Nim to 10 g.xlsx									
11	File WP = C:\Users\malen\Documents\agosto 2023\WP 100 g Nim to 10 g.xlsx									
12	File WP = C:\Users\malen\Documents\agosto 2023\WP 100 g Nim to 10 g.xlsx									
13	Stacking method = Stacked									
14	Method = Gauss Markov									
15	Recalculation = No									
16										
17	Results									
18	Set	ID	Type	Value/g	m-m_N/mg	u/mg (k=1)	Delta m/mg	U(Delta m)/mg (k=2)	En	sigma/ mg
19	Nim	100 Nim	R	99.9999660	-0.0340	0.0030	0.0010	0.0084	0.118	0.00073
20	Nim	100 Nim*	R	100.0001577	0.1577	0.0030	-0.0010	0.0084	-0.118	
21	H15	50 h*	T	49.9999862	-0.0138	0.0015				
22	H15	50 h**	T	49.9999895	-0.0105	0.0015				
23	Nim	20**	T	20.0000223	0.0223	0.0006				
24	Nim	20***	C	20.0000252	0.0252	0.0006	0.0018	0.0042	0.422	
25	Nim	10	T	9.9999779	-0.0221	0.0004				
26	Nim	10*	T	10.0000208	0.0208	0.0004				
27										

Figure 14. Example of the “Results” worksheet

2) “Results_conv_mass”

This worksheet contains:

- the list of the mass standards and the results in terms of conventional mass

An example is given in Fig. 15.

	A	B	C	D	E	F
1	Set	ID	Type	Value/g	m-m_N/mg	u/mg (k=1)
2	Nim	100 Nim	R	99.9999514	-0.0486	0.0030
3	Nim	100 Nim*	R	100.0001431	0.1431	0.0030
4	H15	50 h*	T	49.9999961	-0.0039	0.0015
5	H15	50 h**	T	49.9999987	-0.0013	0.0015
6	Nim	20**	T	20.0000200	0.0200	0.0006
7	Nim	20***	C	20.0000228	0.0228	0.0006
8	Nim	10	T	9.9999761	-0.0239	0.0003
9	Nim	10*	T	10.0000195	0.0195	0.0003
10						
11						
12						

Figure 15. Example of the “Results_conv_mass” worksheet

3) “Weighing program”

This worksheet contains:

- the list of the weights with their characteristics as given in the database file.
The volume shown is that indicated by the User in the database, or calculated (see Appendix A) if the User has provided the density value.

An example is given in Fig. 16.

1	Set	ID	Nom / g	m/g	u(m)/g	V/cm3	u(V)/cm3	hCoG/cm	h/cm	alpha x10^6/°C^-1	u(alpha) x10^6/°C^-1	u(inst)/g	u(m,V)	Ref. Cal.	Cal. Date
2	Nim	100 Nim	100	99,99996502	3E-06	12,5122	0,00013	0,4	0,8	36,6	2	0	0		
3	Nim	100 Nim*	100	100,0001587	3E-06	12,5122	0,00013	0,4	0,8	36,6	2	0	0		
4	H15	50 h*	50	50	1	6,2418	0,00015	0,8	1,6	45	2	0	0		
5	H15	50 h**	50	50	1	6,2423	0,0002	0,8	1,6	45	2	0	0		
6	Nim	20**	20	20	1	2,5019	0,0001327	0,25	0,5	36,6	2	0	0		
7	Nim	20***	20	20,00002343	2E-06	2,502	0,0001327	0,25	0,5	36,6	2	0	0		
8	Nim	10	10	10	1	1,2515	0,00018	0,125	0,25	36,6	2	0	0		
9	Nim	10*	10	10	1	1,2511	0,00018	0,125	0,25	36,6	2	0	0		
10															
11															
12															

Figure 16. Example of the “Weighing Program” worksheet

4) “Weighing Design”

This worksheet contains:

- the table with the design matrix
- the residuals
- the deviations limit
- the values of the residual test
- additional uncertainty (if the recalculation is performed)

An example is given in Fig. 17.

1	A	B	C	D	E	F	G	H	I	J	K	L
1	100 Nim	100 Nim*	50 h*	50 h**	20**	20***	10	10*	Res/mg	dev_lim/mg	Test	
2	1	-1	0	0	0	0	0	0	0.0000	0.0004	true	
3	1	0	-1	-1	0	0	0	0	0.0002	0.0004	true	
4	0	1	-1	-1	0	0	0	0	-0.0002	0.0004	true	
5	0	0	1	-1	0	0	0	0	-0.0001	0.0004	true	
6	0	0	1	0	-1	-1	-1	0	0.0001	0.0004	true	
7	0	0	0	1	-1	-1	0	-1	-0.0001	0.0004	true	
8	0	0	0	0	1	-1	0	0	-0.0000	0.0004	true	
9	0	0	0	0	1	0	-1	-1	0.0000	0.0004	true	
10	0	0	0	0	0	1	-1	-1	-0.0000	0.0004	true	
11	0	0	0	0	0	0	1	-1	0.0001	0.0004	true	
12												

Figure 17. Example of the “Weighing Design” worksheet

5) “Measurements”

This worksheet contains for each weighing equation:

- Diff Bal/mg: mass difference from the balance ($\Delta m'_w$ see Appendix A)
- Corr. Pos : correction in mass due to the positions of the weights on the balance (Δm_E see Appendix A)
- Sens: sensitivity of the balance correct for the buoyancy ($S(1 - \rho_a / \rho_{mAdj})$ Appendix A)
- Delta V/cm³ : volume difference (ΔV equ. 17 Appendix A)
- Rho_a/ kg/m³: air density value (ρ_a see Appendix A)
- Corr Bouy: buoyancy correction (m_b equ. 18 Appendix A)
- hgA/cm: centre of gravity for Group A (hG_A see Appendix A)
- hgB/cm: centre of gravity for Group B (hG_B see Appendix A)
- Delta_h/cm: difference of the centre of gravity ($hG_A - hG_B$ see Appendix A)
- Corr g/mg : correction in mass for the difference of the centre of gravity ($m_N \left[\frac{\Delta g}{g} (hG_A - hG_B) \right]$ Appendix A)
- Dm/mg: mass difference (Δm equ. 10 Appendix A)

An example is given in Fig. 18.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Diff Bal/mg	Corr.pos./mg	Sens	Delta V/cm ³	Rho_a/kg/m ³	Corr Buoy/mg	hgA/cm	hgB/cm	Delta_h/cm	Corr g/mg	Dm/mg	
2	-0,1917	0	0,999787	0,0000	1,1858	0,0000	0,4	0,4	0,000	0,000	-0,1917	
3	-0,0416	0	0,999791	0,0282	1,1508	0,0324	0,4	1,6	-1,200	0,000	-0,0095	
4	0,1497	0	0,999791	0,0282	1,1508	0,0324	0,4	1,6	-1,200	0,000	0,1818	
5	-0,0028	0	0,999789	-0,0005	1,1684	-0,0006	0,8	0,8	0,000	0,000	-0,0034	
6	-0,0233	0	0,999791	-0,0136	1,1568	-0,0158	0,8	0,525	0,275	0,000	-0,0390	
7	-0,0641	0	0,999789	-0,0127	1,1684	-0,0149	0,8	0,525	0,275	0,000	-0,0789	
8	-0,0029	0	0,999791	-0,0001	1,1552	-0,0001	0,25	0,25	0,000	0,000	-0,0030	
9	0,0244	0	0,999791	-0,0007	1,1552	-0,0008	0,25	0,25	0,000	0,000	0,0236	
10	0,0272	0	0,999791	-0,0006	1,1552	-0,0007	0,25	0,25	0,000	0,000	0,0265	
11	-0,0433	0	0,999791	0,0004	1,1529	0,0005	0,125	0,125	0,000	0,000	-0,0428	
12												

Figure 18. Example of the “Measurements” worksheet

6) “Input uncertainties”

This worksheet contains the standard uncertainty contributions for the mass difference (in mg), due to :

- Res. Bal./mg; balance resolution ($\sqrt{2 \frac{res^2}{12}}$ Appendix A)
- Sens./mg : sensitivity of the balance ($\frac{\Delta m_w}{m_s} u(m_s)$ Appendix A)
- Ecc./mg: eccentricity of the balance (u_E Appendix A)
- Lin./ mg: linearity of the balance (u_L Appendix A)
- CoG/mg: centre of gravity correction (u_{CoG} Appendix A)
- m_Diff./mg: mass difference (u_{diff} Appendix A)

- c_V /mg: volume of the standards (square root of the diagonal of $\Psi_{\Delta V}$ Appendix A)
- c_{air} /mg: air density uncertainty (square root of the diagonal of Ψ_{air} Appendix A)
- Additional uncertainty for the recalculation (u_{add} Appendix A)

An example is given in Fig. 19.

	A	B	C	D	E	F	G	H	I
1	Res.Bal./mg	Sens./mg	Ecc./mg	Lin./mg	CoG/mg	m_Diff./mg	c_V /mg	c_{air} /mg	Add./mg
2	4,08248E-05	1,91691E-07	0	0	3,39411E-06	0,0002	0,000219523	0	0
3	4,08248E-05	4,16321E-08	0	0	1,24708E-05	0,0002	0,000324996	1,40838E-05	0
4	4,08248E-05	1,49714E-07	0	0	1,24708E-05	0,0002	0,000324999	1,40838E-05	0
5	4,08248E-05	2,82247E-09	0	0	1,69706E-06	0,0002	0,00029238	2,49993E-07	0
6	4,08248E-05	2,32518E-08	0	0	2,18418E-06	0,0002	0,000347412	6,81785E-06	0
7	4,08248E-05	6,40869E-08	0	0	2,18418E-06	0,0002	0,000383443	6,36574E-06	0
8	4,08248E-05	2,8533E-09	0	0	6,78823E-07	0,0002	0,00021685	4,9999E-08	0
9	4,08248E-05	2,43931E-08	0	0	6,78823E-07	0,0002	0,000331646	3,49993E-07	0
10	4,08248E-05	2,71806E-08	0	0	6,78823E-07	0,0002	0,000331664	2,99994E-07	0
11	4,08248E-05	4,32971E-08	0	0	3,39411E-07	0,0002	0,000293491	1,99995E-07	0
12									

Figure 19. Example of the “Input Uncertainties” worksheet

7) “Additional data”

This worksheet contains the information about the calibration parameters, in the case of no modifications made by the User it corresponds to the values of the “Info” worksheet of the database.

An example is given in Fig. 20.

	A	B	C
1	u dens air/kg/m3	0.0005	
2	Corr dens air	0.5	
3	Corr ref	0.95	
4	Grad g	0.0024	
5	u Grad g	0.0001	
6	u hCoG/cm	0.01	
7	hr%/tdw	tdw	
8			
9			
10			

Figure 20. Example of the “Additional data” worksheet

Recording of subsequent calibrations

Once the results file has been saved by the `SAVE` button, the User can record the results of individual weights on an Excel file, with the aim to monitor the results of subsequent calibrations.

The Excel file will have the name of the database to which the text “_Records” will be appended. For example if the database name is “DB.xlsx” the file with the records will be “DB_Recods.xlsx”.

This file will be saved in the same directory of the database in use.

The results to be saved are indicated by selecting the weights from the “Save” column of the “Results” table on the GUI.

An example is given in Fig. 21

Results								
	Type	Value/g	$m - m_N$ /mg	u /mg (k=1)	Δm /mg	$U(\Delta m)$ /mg	En	Save
Nim 100 Nim	R	99.9999650	-0.0350	0.0030	0.0000	0.0085	0.000	<input type="checkbox"/>
Nim 100 Nim*	T	100.0001566	0.1566	0.0030				<input type="checkbox"/>
H15 50 h*	T	49.9999856	-0.0144	0.0015				<input checked="" type="checkbox"/>
H15 50 h**	T	49.9999889	-0.0111	0.0015				<input checked="" type="checkbox"/>
Nim 20**	T	20.0000220	0.0220	0.0006				<input type="checkbox"/>
Nim 20***	T	20.0000249	0.0249	0.0006				<input type="checkbox"/>
Nim 10	T	9.9999777	-0.0223	0.0004				<input type="checkbox"/>
Nim 10*	T	10.0000207	0.0207	0.0004				<input type="checkbox"/>

Figure 21. Example of the “Additional data” worksheet

After selection, the file is saved with the `RECORD` button

The “Records” worksheet of the Excel file contains one row for each weight, the first two columns “Set” and “ID”, subsequent ones the mass value, uncertainty, Date and Time and File name (the one containing the calibration results), these fields are repeated for each registration.

An example is given in Fig. 22. For easier reading, the names of the weights can be grouped using the Excel Filter function.

	A	B	C	D	E	F	G	H	I	J
1	Set	ID	Mass value/g	unc (k=1)	Date and time	File	Mass value/g	unc (k=1)/mg	Date and tim	File
2	H15	50 h*	49.9999856	0.0015	01-Sep-2023 11	C:\Users\ a 10\WP 100 g Nim to 10 g_Elab1.x	49,999993	0.0015	08-Sep-2023	C:\Users\ a 10\WP 100 g Nim to 10 g_Elab2.xlsx
3	H15	50 h**	49.9999889	0.0015	01-Sep-2023 11	C:\Users\ a 10\WP 100 g Nim to 10 g_Elab1.xlsx				
4	Nim	20**	20.0000220	0.0006	01-Sep-2023 11	C:\Users\ a 10\WP 100 g Nim to 10 g_Elab1.xlsx				
5										
6										

Figure 22. Example of the “Records” worksheet

5 Correlations

5.1 Correlations between mass reference standards

The correlation coefficient between two standards m_i and m_j is evaluated by

$$r(m_i, m_j) = \frac{u(m_i, m_j)}{u(m_i)u(m_j)}. \quad (1)$$

where $u(m_i)$ and $u(m_j)$ are the uncertainties of m_i and m_j , and $u(m_i, m_j)$ is the covariance between m_i and m_j .

In general, the covariance is given by the uncertainty of type B in common, associated with calibration, to the weights m_i and m_j , with u_{com} being the common uncertainty, then

$$u(m_i, m_j) = u_{\text{com}}^2. \quad (2)$$

In the case of calibration with the Gauss-Markov method, the covariance matrix of the results obtained is provided, so that the correlation between standards can be derived from the covariance matrix.

5.2 Correlation between volume and mass of the reference standards

The correlation between the mass and the volume $r(m, V)$ of a mass standard arises from the calibration of the mass standard. It is a parameter that can be useful for the reference standards, and whose value is calculated by

$$r(m, V) = \frac{u(m, V)}{u(m)u(V)}, \quad (3)$$

where $u(m, V)$ is the covariance between m and V and $u(V)$ and $u(m)$ are the uncertainties for the volume and for the mass value.

The covariance $u(m, V)$ depends on the calibration of the reference and is determined on the basis of the previous calibration from which the value m was obtained [5]

$$u(m, V) = \rho_{a1} u^2(V) \quad (4)$$

Where ρ_{a1} is the air density at the time of the calibration of the mass standard

For example:

- $u(V) = 0,020 \text{ cm}^3$
- $u(m) = 0,030 \text{ mg}$
- $\rho_{a1} = 1,160 \text{ kg/m}^3$

Therefore $u(m, V) = 4,64 \cdot 10^{-4} \text{ mg cm}^3$ and $r(m, V) = 0.773$.

This term plays an important role in the calculation of uncertainty of mass standards [6-8], in balance calibration [9,10] and in force metrology [11], but also for the propagation of uncertainty between mass and

conventional mass [5]. In practice, it makes it possible to consider that the uncertainty associated with the mass already contains the uncertainty contribution due to its volume uncertainty.

Considering this correlation avoids overrating the uncertainty when this standard is used as reference, especially in cases where the uncertainty associated with the volume (or density) of the reference is non-negligible.

The correlation $r(m, V)$ leads to the correlation between m_{Ref} and Δm_i . This information can be used in the Gauss-Markov method for the implementation of the covariance matrix of input quantities. If the correlation is not calculated and entered into the “Data” worksheet, the uncertainties of the results will be overrated, in particular those relating to conventional mass values. More details are given in Appendix A and Example C3.

6 Mass and conventional mass value

The software has been implemented in accordance with the (true) mass value calculations, however, by conversion, it also provides results expressed in conventional mass m_c by

$$m_c = \frac{m}{1 - \frac{\rho_0}{\rho_c}} - \rho_0 V, \quad (5)$$

where $\rho_0 = 1.2 \text{ kg/m}^3$ and $\rho_c = 8\,000 \text{ kg/m}^3$.

Expression (5) is equivalent to the familiar expression using densities rather than volumes [8]

$$m_c \left(1 - \frac{\rho_0}{\rho_c}\right) = m \left(1 - \frac{\rho_0}{\rho}\right). \quad (6)$$

By taking into account the correlation between mass and volume, the associated uncertainty $u(m_c)$ is given by (see ref. [5])

$$u(m_c) = \sqrt{u^2(m) + [(\rho_{a1} - \rho_0)^2 - \rho_{a1}^2]u^2(V)}, \quad (7)$$

where ρ_{a1} is the air density at the time of the calibration. Since the determination of the value of a standard can be derived from a system of weighing equations, the air density considered in equation (7) is the average value during the whole measurement.

This assumption could lead to slight differences in the calculations in the case where the value of an unknown mass is derived from a one-to-one calibration and the mean value of the air densities does not correspond to the air density of that one-to-one calibration. However, the air density variations in a mass laboratory are small, so that the corresponding variation in $u(m_c)$ is always negligible compared with its value.

From equation (7), it can be noted that the difference between $u(m)$ and $u(m_c)$ can be significant when the uncertainty for the volume is non-negligible, which usually occurs when the volume (or density) of the mass standard is not measured, but only approximately estimated.

As already said, the software is implemented for the calculations in terms of mass, so that the values of the reference standards have to be provided in terms of mass. However, the User can use this software even with mass standards of which only the conventional mass value is known, the method is to convert the conventional mass value and the associated uncertainties in term of mass.

It is important that the uncertainty propagation be performed considering the correlation between mass and volume. The conversion of the value from m_c to m is easily calculated using the equations (5) or (6), this value will be entered in the database.

Regarding uncertainty, two steps are necessary:

- Convert $u(m_c)$ to $u(m)$
- Evaluate the correlation $r(m, V)$

both of which will be entered in the database.

The uncertainty propagation is performed by

$$u(m) = \sqrt{u^2(m_c) - [(\rho_{a1} - \rho_0)^2 - \rho_{a1}^2]u^2(V)}, \quad (8)$$

where ρ_{a1} is the air density at the time of the calibration of the standard.

The correlation coefficient $r(m, V)$ is evaluated as described in section 5.2.

An example of the use of a reference mass standard calibrated in term of conventional mass is given in example C3.

7 Use of software in practice

At the start-up, the software loads the database file used in the previous use, the name appears in the file name cell.

The User can continue to use the already loaded file, or load another database file.

The database to be used can only be chosen at the start-up of the software. If it is necessary to change the database after starting to load the worksheet "WP", the software must be restarted.

After having verified that the database file is correct, the file containing the weighing plan can be loaded. If the loaded file contains the worksheets "WP", "WD" and "Meas" (and if necessary also the "Pos" and "Corr" worksheets) all data will be loaded for the processing. This allows the User to prepare routine weighing schemes, which will be repeated over time by changing only the measurements on the "Meas" worksheet.

Otherwise, subsequently upload the file with the worksheet "WD" and then the file with the worksheet "Meas" (if necessary, add the "Pos" and "Corr" worksheets). This allows the User to process systems of equations by keeping the weighing plan unchanged and simply modifying the design matrix and the measurements file. The buttons for loading these files are enabled in sequence. For example, it is not possible to first load the file with worksheet "Meas" and then the file with worksheet "WD".

The User always has the possibility of loading a new weighing plan, at any time.

Once the file has been loaded with the "WP" worksheet, the User has the option of setting the role of the standards: Reference, Check or Test. This operation can also be performed at any time by the User changing the values of the column "Type" of the "Results" table of the GUI.

Once all data are loaded, the elaboration button is enabled.

Before the data processing, the User must check the settings:

- “Gauss-Markov” or “Lagrange multipliers” calculation method
- grouping method (only if the “Pos” worksheet has not been loaded)
- calibration parameters

After processing, the results are shown in the “Results” table, and the residuals of the measurements are shown in the “Design matrix” table.

Other information is shown regarding the residual test, the calculation of E_n for the reference and check standards.

At this point the User can save the results, or change the settings for a new elaboration:

- change the calculation method (simply by selecting a different statistical model)
- change the role of the standards (simply by editing the “Results” table)
- change the parameters (simply by editing the cells, after deselecting or changing the statistical model selection)
- perform a recalculation taking into account the residuals obtained

After the file has been saved, the User can change again the settings and repeat the elaboration.

Once the results have been saved, the User can select the results of the weights to be saved in the file with the subsequent calibrations.

During the use, error or warning messages may appear (see Appendix B), in which case, to continue to use the software, the User should acknowledge the message by pressing the OK button. If the User does not acknowledge the message correctly, it may happen that the software will stop, in which case check that the window with the message is not hidden by the GUI window.

The algorithms developed in the software are based on processing the weighing equations defined by the design matrix provided by the User. In this way, the application of the software can extend from the most complicated problems involving the design matrices used to realize the mass scale, to the simplest calibrations by one-to-one comparison. In the case of one-to-one calibration with several reference standards of different nominal values, the processing must be carried out by Gauss-Markov method.

Below there are some examples of calibration, with the design matrices used.

- Calibration of two weights of 1 kg (A and B) by two reference standards (Ref1 and Ref2), and redundant measurements to improve the robustness of the calibration results:

1 kg Ref 1	1 kg Ref 2	1 kg A	1 kg B
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	1	0	-1
0	0	1	-1

As two references are involved, only the Gauss-Markov method can be used.

- Calibration of a set of weights (1 kg A, 500 g A and 200 g A and 200 g B) by the one-to-one method:

1 kg Ref	500 g Ref	200 g Ref	1 kg A	500 g A	200 g A	200 g B
1	0	0	-1	0	0	0
0	1	0	0	-1	0	0
0	0	1	0	0	-1	0
0	0	1	0	0	0	-1

As three references are involved, only the Gauss-Markov method can be used.

- Calibration of two weights of 500 g with a reference of 1 kg:

1 kg Ref	500 g A	500 g B
1	-1	-1
0	1	-1

- Calibration by subdivision, from one reference of 1 kg, to 100 g:

1 kg Ref	500 g A	200 g A	200 g B	100 g A	100 g B
1	-1	-1	-1	-1	0
0	1	-1	-1	0	-1
0	0	1	0	-1	-1
0	0	1	-1	0	0
0	0	0	0	1	-1

- Calibration by multiplication, up to 5 kg, from two references of 1 kg:

1 kg Ref 1	1 kg Ref 2	2 kg A	2 kg B	5 kg A
1	1	-1	0	0
0	0	1	-1	0
1	-1	1	-1	0
0	1	1	1	-1
1	0	1	1	-1

As two references are involved, only the Gauss-Markov method can be used.

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Appendix A: Statistical models

The process of establishing the mass of each single standard in a set is often called multiplication, when referring to multiples of the kilogram, or subdivision for submultiples. The latter is the most challenging part of the procedure in realising the mass scale. The process is based on the determination of the mass difference between weights, or groups of weights, of equal nominal value [13].

With this method, for each equation a standard or a group of standards having sum $m_A = \sum m_i$ is compared with another standard or group of standards $m_B = \sum m_j$, where m_A and m_B have the same nominal value m_N . For each comparison a weighing equation is obtained, from which the mass difference Δm_i is derived.

Usually, the system of equations for the realization of the mass scale is overdetermined, that is, the number of linearly independent equations is greater than the number of weights to be calibrated.

However, the problem can also be solved when only one reference is given and the number of equations is equal to the number of the unknowns. In the case that the number of equations is less than the number of unknowns, the problem can be also solved if more than one standard is involved, in the appropriate way.

This method can also be used when comparing several weights of the same nominal value, by several one-to-one calibrations.

To estimate the unknown values, the software uses the Weighted Least Squares by Lagrange Multipliers [14] and the Gauss-Markov approach [15], which are able to provide the results taking into account the different uncertainties of the input quantities.

Implementation of the system of equations in matrix form

The system of equations can be conveniently represented in matrix form as:

$$\mathbf{X}_w \mathbf{m} = \mathbf{y}_w + \boldsymbol{\varepsilon}, \quad (9)$$

where:

- \mathbf{X}_w is a matrix (the design matrix) whose entries are +1, -1 or 0, according to the role played by each weight in the formation of the two groups of weights to be compared (defined by the worksheet “WD”)
- $\mathbf{m}^T = [m_1 m_2 \dots m_p]$ is the column vector of the parameters of the system, that is, the values of the weights (defined by the worksheet “WP”)
- $\mathbf{y}_w^T = [\Delta m_1 \Delta m_2 \dots \Delta m_q]$ is the column vector of the mass differences between the two groups of standards involved in each equation (evaluated with the worksheet “Meas”).
- $\boldsymbol{\varepsilon}$ is the vector of (unknown) measurement errors

Each mass difference Δm_i for a given weighing equation with nominal mass value m_N is determined by the weighing model, which can be expressed as (for ease of presentation the subscript i has been dropped)

$$\Delta m = \delta m \left(1 - \frac{\rho_a}{\rho_{mAdj}} \right) + \rho_a \left(\sum V_i - \sum V_j \right) + m_N \left[\frac{\Delta g}{g} (hG_A - hG_B) \right], \quad (10)$$

where:

- δm is the mass difference determined by the balance
- ρ_a is the air density

- ρ_{mAdj} is the density of the adjustment mass
- $\sum V_i$ is the sum of the volumes of the groups m_A
- $\sum V_j$ is the sum of the volumes of the groups m_B
- $\frac{\Delta g}{g}$ is the gradient of the acceleration of gravity
- hG_A, hG_B are the centre of gravity for the group m_A and m_B , respectively

In the following, it is shown how the software calculates the various components.

Evaluation of the mass difference determined by the balance δm

The difference of mass δm determined by the balance is given by

$$\delta m = \Delta m_w S, \quad (11)$$

where:

- Δm_w is the corrected difference measured by the balance (equation 13)
- S is the sensitivity of the balance.

The sensitivity S of the balance is

$$S = \frac{m_s}{\Delta m_s}, \quad (12)$$

where m_s is the value of the adjustment mass or sensitivity mass (worksheet “Balances” in g) and Δm_s the corresponding value read by the balance when the adjustment mass is loaded (worksheet “Balances” in g).

In the case of eccentricity problems due to the positions of the weights, it is recommended to correct the value of the measured difference.

If this effect Δm_E is known to be a function of the balance in use and the position correction is given, the value is corrected as

$$\Delta m_w = \Delta m'_w + \Delta m_E, \quad (13)$$

where $\Delta m'_w$ is the mass difference measured by the balance (worksheet “Meas” in mg).

The values of $m_s, \Delta m_E, \Delta m_s, \rho_{Adj\ mass}$ depend on the balance, the values are in the worksheet “Balances”. If the balance is specified in the worksheet “Meas”, these calculations are automatic.

In case the balance is not specified

$$S = 1, \Delta m_E = 0 \text{ and } \rho_{mAdj} = 8\,000 \text{ kg/m}^3, \quad (14)$$

so that the first term of equation (10) is

$$\delta m \left(1 - \frac{\rho_a}{\rho_{mAdj}} \right) = \Delta m'_w \left(1 - \frac{\rho_a}{8000} \right). \quad (15)$$

Evaluation of the air buoyancy correction

For each weighing equation the volume of each standard V is evaluated at the temperature of measurement t as

$$V = V_{20}[1 - \alpha(t - 20)], \quad (16)$$

where V_{20} is the volume at 20 °C, and α the expansion coefficient. The two values are provided in the worksheet “Data”.

As the temperature of a calibration mass laboratory is usually stable, the temperature value t is determined as the average of the temperatures given for the weighing equations.

The vector of the volume difference ΔV can be obtained as

$$\Delta V = X_w V, \quad (17)$$

where $V^T = [V_1, V_2, \dots, V_p]$ is the column vector of the values of the volumes of the weights (equation 16), associated with the vector m^T .

In matrix form the air buoyancy correction vector is

$$m_b = \rho_a X_w V, \quad (18)$$

where ρ_a is a diagonal matrix, in which on the diagonal there are the air densities for each weighing equation.

In case the User provides only the density of the weight ρ_i , and the uncertainty $u(\rho_i)$, the corresponding volume is evaluated as $V_i = m_i/\rho_i$, where m_i is the previously estimated mass value. The associated uncertainty is estimated by $u(V_i) = V_i u(\rho_i)/\rho_i$. The value m_i is the mass value in the worksheet “Data”.

In case the User provide both volume and density of the standard, only the volume is considered for the calculation. The density and the volume of each weight are provided in the worksheet “Data”.

The air density can be provided by the User, or can be evaluated by the CIPM 2007 equation [4] from the environmental parameters (worksheet “Meas”).

In case the User provides the air density and also the environmental parameters, only the air density value provided by the User will be used in the calculation.

Evaluation of the difference of centre of gravity

The User can provide the center of gravity and height for each weight (worksheet “Data”), if not provided the assigned values are null.

The software can also evaluate the center of gravity of the group of weights. The center of gravity of each group can be evaluated from the center of gravity and height of the standard of each weight involved in the group, and taking into account how the group was formed, that is, with stacked or side-by-side standards.

The centre of gravity of a group of k weights is given by

$$hG = \frac{m_{N1}(hG_1+hB_1)+m_{N2}(hG_2+hB_2)+\dots+m_{Nk}(hG_k+hB_k)}{\sum m_{Ni}}, \quad (19)$$

where

- m_{Ni} is the nominal mass of each weight (worksheet “Data”)
- hG_i is the height of centre of gravity of each weight (worksheet “Data”)
- hB_i is the height at which the base of the weight is located

In case of side-by-side method:

$$hG = \frac{\sum m_{Ni} hG_i}{\sum m_{Ni}}. \quad (20)$$

In case of stacked method:

Considering that the base of the stack is the mass m_1 to which mass m_2 is superimposed and so on, up to m_k ,

$$hG = \frac{m_{N1} hG_1+m_{N2}(hG_2+h_1)+\dots+m_{Nk}(hG_k+h_{k-2}+h_{k-1})}{\sum m_{Ni}}, \quad (21)$$

where h_i is the height of the i th weight (worksheet “Data”).

The two grouping methods, “side-by side” and “stacked”, if used for all the weighings, can be chosen from the GUI.

If the grouping method does not relate only to “side-by-side” or “stacked” weights for all the weighings, the User can provide the matrix which defines the “Custom” method by the worksheet “Pos”, and an algorithm of the software calculates the centre of mass of each group considering the position of the weights as described in the worksheet “Pos” (see section 4.4) by equation (19).

Statistical Model

In order to solve the system of equations (9), it is necessary to provide a constraint, which defines the value of at least one reference standard. This additional information modifies the model (9), by changing the matrix X_w and the vector y_w .

The software can deal with this problem using two estimators:

- Gauss-Markov method
- Lagrange Multipliers method

As the results are strongly influenced by the values of the residuals, the software provides an additional robust method based on residual analysis.

The Gauss-Markov method

In order to solve the system of equations (9), it is necessary to provide the value of at least one standard, that will be the reference. In matrix form [15]

$$\mathbf{A}\mathbf{m} = \mathbf{R} , \quad (22)$$

where \mathbf{A} is a matrix which select the reference standards from the vector \mathbf{m} , and $\mathbf{R}^T=[m_{R1},m_{R2}...m_{Rn}]$ is the vector with the associated values of the references mass standards.

For example, if we want to set as reference standards the first two weights of the vector \mathbf{m} : $m_1 = m_{R1}$ and $m_2 = m_{R2}$, the matrices will be

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \end{bmatrix} , \quad (23)$$

$$\mathbf{R} = \begin{bmatrix} m_{R1} \\ m_{R2} \end{bmatrix} \quad (24)$$

and

$$\begin{bmatrix} 1 & 0 & 0 & \dots \\ 0 & 1 & 0 & \dots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} m_{R1} \\ m_{R2} \end{bmatrix} . \quad (25)$$

In order to consider this information in the system of equations, equation (9) becomes

$$\mathbf{X}\mathbf{m} = \mathbf{y} + \boldsymbol{\varepsilon} , \quad (26)$$

where \mathbf{X} is a composition of two matrices

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_w \\ \mathbf{A} \end{bmatrix} \quad (27)$$

and \mathbf{y} is a composition of two vectors

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_w \\ \mathbf{R} \end{bmatrix} . \quad (28)$$

This matrix construction is performed automatically by the software on the basis of User selection of the reference standards to be used. The reference standards are defined in the table “Results” of the GUI, with the letter R.

The results $\hat{\mathbf{m}}$ are evaluated as

$$\hat{\mathbf{m}} = (\mathbf{X}^T \boldsymbol{\Psi}_y^{-1} \mathbf{X})^{-1} \mathbf{X}^T \boldsymbol{\Psi}_y^{-1} \mathbf{y} , \quad (29)$$

where $\boldsymbol{\Psi}_y$ is the covariance matrix associated to the vector \mathbf{y} .

The covariance matrix $\boldsymbol{\Psi}_{\hat{\mathbf{m}}}$ for the results $\hat{\mathbf{m}}$ will be

$$\boldsymbol{\Psi}_{\hat{\mathbf{m}}} = (\mathbf{X}^T \boldsymbol{\Psi}_y^{-1} \mathbf{X})^{-1} . \quad (30)$$

The square roots of the diagonal terms of $\boldsymbol{\Psi}_{\hat{\mathbf{m}}}$ are the standard uncertainties associated with the vector $\hat{\mathbf{m}}$ and the off-diagonal terms are the covariances between the mass standards.

The real-mass estimates are converted to conventional mass estimates according to the method described in section 6.

Determination of the covariance matrix Ψ_y

The vector y is a function of several quantities, as shown by model (10). The corresponding covariance matrix Ψ_y is obtained as the sum of several other covariance matrices as shown in the paper by Bich et al. [16]:

$$\Psi_y = \Psi_{\delta m} + \Psi_{\text{air}} + \Psi_{\Delta V} + \Psi_{RS}, \quad (31)$$

where:

- $\Psi_{\delta m}$ covariance matrix for the mass differences measured by the balance
- Ψ_{air} covariance matrix for air density
- $\Psi_{\Delta V}$ covariance matrix for volume differences
- Ψ_{RS} covariance matrix for the reference standards.

These covariance matrices are evaluated by applying the propagation of uncertainties as described in [16], considering the possible correlations, such as those between reference standards and those due to the determination of air density. In this software $\Psi_{\delta m}$ and $\Psi_{\Delta V}$ are diagonal, that is without covariance terms.

In addition to the mentioned covariance matrices, the model used in this software has been improved by taking into account the correlations between the values m and V of the reference standards $r(m,V)$, which leads to correlations between the value of the reference standards and the measured mass differences in which the reference standards are involved. For this reason the covariance matrix $\Psi_{m,\Delta m}$ is evaluated. The term $r(m,V)$ is described in the paper by Malengo et al. [5], and plays an important role in uncertainty evaluation, as it considers the air buoyancy uncertainty already contained in the uncertainty of the mass reference.

In the following, it is shown how the software implements the covariance matrices.

- Evaluation of $\Psi_{\delta m}$

It is assumed that the mass differences δm are uncorrelated and that the uncertainty contributions depend on: resolution of the balance res , repeatability of the measurements σ , eccentricity u_E , sensitivity, linearity u_L and difference of centre of gravity u_{COG} .

The evaluation of the uncertainty contributions can be performed in two different ways, depending on whether the balance used was specified or not (column “Balance” in worksheet “Meas”).

If the balance is specified: the terms res , u_E , u_L , $u(m_s)$ and m_s are provided from worksheet “Balances”, the contribution for each weighing is evaluated as

$$u_{\Delta m}^2 = u_{diff}^2 + 2 \frac{res^2}{12} + u_E^2 + u_L^2 + \Delta m_w^2 \frac{u^2(m_s)}{m_s^2} + u_{COG}^2, \quad (32)$$

where

- u_{diff} is the standard deviation of the mean value of δm , which can be evaluated as

$$u_{diff} = \sqrt{\frac{\sigma^2}{n}} \quad , \quad (33)$$

with n the number of repetitions of the measurement. u_{diff} is provided by the User from the column “u Diff mg” of the worksheet “Meas”.

If necessary, the User could include in u_{diff} any additional uncertainty contributions such as short-term instability associated to the weights.

The uncertainty associated to the correction due to the centre of gravity is given by

$$u_{COG} = \sqrt{2u^2(hG)\left(\frac{\Delta g}{g}\right)^2 m_N + u^2\left(\frac{\Delta g}{g}\right)m_N^2(hG_A - hG_B)^2} \quad , \quad (34)$$

where

- $u(hG)$ is the uncertainty of the height of centre of gravity of the weights (worksheet “Info”)
- $u\left(\frac{\Delta g}{g}\right)$ is the uncertainty for the gradient of g (worksheet “Info”)

If the balance is not specified:

$$u_{\Delta m_i}^2 = u_{diff}^2 + u_{COG}^2 \quad , \quad (35)$$

that is, only the contribution given in the column “u Diff mg” of the worksheet “Meas” and u_{COG} are considered. It is the User's responsibility to evaluate the uncertainty due to the balance in the appropriate way, and insert the value in column “u Diff mg”.

The covariance matrix is given by

$$\Psi_{\delta m} = \begin{pmatrix} u_{\Delta m1}^2 & 0 & 0 & 0 \\ 0 & u_{\Delta m2}^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & u_{\Delta mq}^2 \end{pmatrix} \quad . \quad (36)$$

At the end of the processing, the User has the option of repeating the calculation by the recalculation function, by adding an additional uncertainty u_{add} [12] that depends on the value of the residual test and the residual obtained (as described in section 4.8), so that the values on the diagonal become $u_{\Delta m_i}^2 + u_{add i}^2$.

- Evaluation of Ψ_{air}

As in general the air buoyancy correction is given by

$$m_{bi} = \rho_a(V_j - V_k) \quad . \quad (37)$$

Elements on the diagonal of Ψ_{air} are

$$u^2(\rho_a)(V_j - V_k)^2 \quad (38)$$

where $u(\rho_a)$ is the air density uncertainty.

Usually air densities are determined by the same instruments, therefore the air density uncertainty is always the same for all the measurements and in addition the air density measurements are correlated.

Considering for example

$$m_{b1} = \rho_{1a}(V_j - V_k) \text{ and } m_{b2} = \rho_{2a}(V_j - V_z),$$

m_{b1} and m_{b2} are correlated for the air density measurements, which is evaluated by

$$u(m_{b1}, m_{b2}) = (V_j - V_k)(V_j - V_z)u(\rho_{1a}, \rho_{2a}), \quad (39)$$

where $u(\rho_{1a}, \rho_{2a})$, the common variance for ρ_{1a}, ρ_{2a} , which is evaluated as $u(\rho_{1a}, \rho_{2a}) = ru(\rho_{1a})u(\rho_{2a})$,

where the correlation r , depends on the instruments (usually about $0,5 < r < 0,9$).

In this software it is assumed that the uncertainties associated to the air density measurements are the same for all measurements

$$u(\rho_a) = u(\rho_{1a}) = u(\rho_{2a}). \quad (40)$$

The correlation value and the air density uncertainty are provided by the worksheet "Info".

In matrix form we obtain:

$$\mathbf{Cov}_a = \begin{pmatrix} u^2(\rho_a) & ru(\rho_a)u(\rho_a) & ru(\rho_a)u(\rho_a) & ru(\rho_a)u(\rho_a) \\ \dots & u^2(\rho_a) & ru(\rho_a)u(\rho_a) & ru(\rho_a)u(\rho_a) \\ \dots & \dots & u^2(\rho_a) & ru(\rho_a)u(\rho_a) \\ SYM & \dots & \dots & u_N^2(\rho_a) \end{pmatrix}, \quad (41)$$

for the law of propagation of the uncertainties

$$\mathbf{\Psi}_{\text{air}} = \begin{pmatrix} dV_1 & 0 & 0 & 0 \\ 0 & dV_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & dV_N \end{pmatrix} \mathbf{Cov}_a \begin{pmatrix} dV_1 & 0 & 0 & 0 \\ 0 & dV_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & dV_N \end{pmatrix}, \quad (42)$$

with ΔV the diagonal matrix with the volume differences (equation 17).

- Covariance matrix for the volume $\mathbf{\Psi}_{\Delta V}$

As in general

$$m_{bi} = \rho_a(V_j - V_k), \quad (43)$$

elements on the diagonal of $\mathbf{\Psi}_{\Delta V}$ are

$$u^2(V_j - V_k)\rho_a^2. \quad (44)$$

The covariance matrix is given by

$$\Psi_{\Delta V} = \mathbf{X}_{\rho a} \mathbf{CovV} \mathbf{X}_{\rho a}^T, \quad (45)$$

where

$$\mathbf{X}_{\rho a} = \begin{pmatrix} \rho_{1a} & 0 & 0 & 0 \\ 0 & \rho_{2a} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \rho_{Na} \end{pmatrix} \mathbf{X}_W \quad (46)$$

And \mathbf{CovV} is the covariance matrix for the volumes of the weights. As the software considers the correlations between volumes to be null

$$\mathbf{CovV} = \begin{pmatrix} u^2(V_1) & 0 & 0 & 0 \\ 0 & u^2(V_2) & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & u^2(V_m) \end{pmatrix}. \quad (47)$$

The standard uncertainty $u(V_i)$ of each weight is evaluated as

$$u(V_i) = \sqrt{u^2(V_{20i}) + u^2(\alpha_i)V_i^2(t - 20)^2}. \quad (48)$$

The contribution for the temperature uncertainty is considered negligible.

- Covariance matrix for the reference standards Ψ_{RS}

In general, when using the Gauss-Markov method, it is possible to use several references with values m_{Ri} and uncertainties $u(m_{Ri})$.

Elements on the diagonal of the matrix Ψ_{RS} are the variance $u^2(m_{Ri})$.

Considering the correlations between the reference standards, covariance terms should be taken into account. For example, if there are two reference standards, considering the correlation between m_{R1} and m_{R2} the covariance matrix is

$$\Psi_{RS} = \begin{pmatrix} u^2(m_{R1}) & u(m_{R1}, m_{R2}) & \dots \\ u(m_{R1}, m_{R2}) & u^2(m_{R2}) & \dots \\ \dots & \dots & u^2(m_{Rm}) \end{pmatrix}. \quad (49)$$

The term $u(m_{R1}, m_{R2})$ is the covariance between m_{R1} and m_{R2} .

The uncertainty of the mass references is evaluated by

$$u(m_{Ri}) = \sqrt{u^2(m_i) + u_{inst}^2} \quad (50)$$

where $u(m_i)$ is the uncertainty due to the calibration and u_{inst} the uncertainty contribution due to the instability. These values are provided by database file, worksheet "Data".

If the User also include in the value $u(m_i)$ the contribution due to the instability, is not necessary to use the value u_{inst} .

The covariance terms are evaluated by

$$u(m_{Ri}, m_{Rj}) = r_{i,j} u(m_{Ri}) u(m_{Rj}) . \quad (51)$$

The correlation value can be provided by the worksheet “Info”, by the GUI, or by the worksheet “Corr”, see section 4.3.

- Covariance matrix $\Psi_{m,\Delta m}$

In the case that the User has provided the correlation $r(m,V)$ between the mass and volume of the reference standards, it is possible to calculate the covariance $u(m,V)$

$$u(m, V) = r(m, V)u(m)u(V) . \quad (52)$$

The correlation between mass and volume of a reference standard, also correlate the mass differences in which the reference is involved and the mass value of the reference, so that the covariance term $u(\Delta m, m)$ is then taken into account

$$u(\Delta m, m) = (\pm)\rho_a u(m, V) , \quad (53)$$

where the sign corresponds to the sign of the design matrix associated to the mass reference.

For the covariance matrix $\Psi_{m,\Delta m}$ the number of rows corresponds to the number of row of the design matrix X_w and the number of column corresponds to the number of the reference standards. The matrix $\Psi_{\Delta m,m}$ is formed with non-zero terms only in correspondence of the row of the matrix X_w in which the reference is involved and the column of the matrix Ψ_{RS} corresponding to the reference.

Implementation of the matrix Ψ_y

In conclusion, with no correlation between the mass of the reference standards and the measured differences, Ψ_y is a composition of matrices

$$\Psi_y = \begin{pmatrix} \Psi_{\delta m} + \Psi_{air} + \Psi_{\Delta V} & \mathbf{0} \\ \mathbf{0} & \Psi_{RS} \end{pmatrix} , \quad (54)$$

with correlation between the reference standards and the measured differences

$$\Psi_y = \begin{pmatrix} \Psi_{\delta m} + \Psi_{air} + \Psi_{\Delta V} & \Psi_{m,\Delta m} \\ \Psi'_{m,\Delta m} & \Psi_{RS} \end{pmatrix} . \quad (55)$$

The Lagrange Multipliers method

In order to solve the system of equations (9), a constraint, that is the mass value of one reference standard is required, in matrix form

$$\mathbf{A}\mathbf{m} = \mathbf{R} . \quad (56)$$

Since only one reference standard needs to be defined, the matrix \mathbf{A} is a row vector that selects the chosen reference, and \mathbf{R} is the value of the reference standard.

For example, if the reference is the first weight of the vector \mathbf{m} , that is $m_1 = m_{R1}$

$$\mathbf{A} = [1 \ 0 \ 0 \ \dots] . \quad (57)$$

The value of the reference m_{R1} is added to the end of the vector \mathbf{y}_w appropriately weighed (see equ. 61 and 63), in addition the formal parameter λ , which is the Lagrange multiplier, is added to the end of the vector \mathbf{m} [14].

As the measured mass differences can have different accuracies, it is necessary for the weighting equations to be mathematically weighted before least-squares adjustment.

For this purpose, the diagonal weight matrix \mathbf{W} is introduced [14,17], where the terms on the diagonal are evaluated as

$$w_{ii} = \left(\frac{\sigma_0}{s_i} \right)^2 , \quad (58)$$

where s_i is the standard uncertainty associated to the mass difference δm of each weighing equation, that is the terms of equation (32), and σ_0 is the normalisation factor defined by:

$$\sigma_0^2 = \frac{1}{\sum_1^p \frac{1}{s_i^2}} . \quad (59)$$

This approach is different from the method in [17], since for s_i , not only the standard deviations of the mean values of δm are considered, but also the contributions of uncertainty in terms of resolution, sensitivity, linearity and eccentricity of the balances.

The new matrices are

$$\mathbf{X}' = \sqrt{\mathbf{W}}\mathbf{X}_w \quad (60)$$

and

$$\mathbf{y}' = \sqrt{\mathbf{W}}\mathbf{y}_w . \quad (61)$$

The solution is given by

$$\hat{\mathbf{m}} = \mathbf{L}\mathbf{y} \quad (62)$$

where

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}' \\ \mathbf{m}_{Ref} \end{bmatrix} , \quad (63)$$

$$L = \begin{bmatrix} X'^T X' & A' \\ A & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} X'^T & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} . \quad (64)$$

Uncertainty evaluation

The covariance matrix of the solutions $\hat{\mathbf{m}}$ is calculated according to the general law of uncertainties propagation, as follows

$$\Psi_{\hat{\mathbf{m}}} = L' \Psi_{\mathbf{y}'} L'^T , \quad (65)$$

where $\Psi_{\mathbf{y}'}$ is the covariance matrix associated to the vector of the mass differences \mathbf{y}' . For this method these matrices are

$$\Psi_{\mathbf{y}'} = \Psi_{\delta m} \quad (66)$$

and

$$L' = C X_w^T W , \quad (67)$$

where the matrix C is extracted from the matrix [14]

$$\begin{pmatrix} C & \mathbf{h} \\ \mathbf{h}' & \mathbf{0} \end{pmatrix} = \begin{bmatrix} X'^T X' & A' \\ A & \mathbf{0} \end{bmatrix}^{-1} . \quad (68)$$

As $\Psi_{\mathbf{y}'}$ does not contain all the uncertainty contributions, with the Lagrange multipliers an incomplete variance-covariance matrix can be obtained.

The problem is solved by combining all uncertainty contributions, as follow:

- *Weighing contribution*

The standard uncertainties are the square roots of the diagonal elements of the matrix $\Psi_{\hat{\mathbf{m}}}$

$$\Psi_{\hat{\mathbf{m}}} = C X_w^T W \Psi_{\delta m} (C X_w^T W)^T .$$

- *Reference standard contribution*

The uncertainty contribution for each mas standard m_i is evaluated as

$$u(m_i) = h_i u(m_R) \quad (69)$$

where

$$h_i = \frac{m_i}{m_R} . \quad (70)$$

The value of h_i are the elements in the last column of the solution matrix L .

- *Air buoyancy uncertainty contribution*

The uncertainties for the air densities are evaluated as the square root of the terms on the diagonal of the matrix

$$\Psi_{air} = CX_w^T W \Delta V Cov_a (CX_w^T W \Delta V)^T . \quad (71)$$

The uncertainties for the volumes are evaluated as the square root of the terms on the diagonal of the matrix

$$\Psi_{\Delta V} = CX_w^T W X_{\rho a} Cov V (CX_w^T W X_{\rho a})^T . \quad (72)$$

Validation of the results

In order to check the goodness of the fit, the GUI shows both the standard deviation of the fit

$$\sigma = \sqrt{\frac{(y_w - X_w \hat{m})^T (y_w - X_w \hat{m})}{q - p + n}} \quad (73)$$

and also the residual r_i for each weighing equation, that is the vector

$$\mathbf{r} = \mathbf{y}_w - \mathbf{X}_w \hat{\mathbf{m}} . \quad (74)$$

This allows the User to validate the results and detect any possible mistakes in the measurements.

As an auxiliary tool, the result of the residual test is also shown, which checks that the residual value for each mass difference r_i is less than a limit threshold Lim. Since the residual is expected to be within the weighing uncertainty, this threshold is evaluated as expanded uncertainty, considering the contributions due to the resolution of the balance u_{res} and the weighing uncertainty u_{diff} .

$$\text{Lim} = 2 \sqrt{u_{res}^2 + u_{diff}^2} . \quad (75)$$

If the balance is not specified, only u_{diff} is considered. In case of recalculation, also the additional uncertainty u_{add} is considered

$$\text{Lim} = 2 \sqrt{u_{res}^2 + u_{diff}^2 + u_{add}^2} . \quad (76)$$

For check standards, and reference standards the calculation of the normalised error E_n is also carried out. The difference Δm (table “Results”) between the calculated mass value m and the value provided by the User m_p (worksheet “Data”) is calculated, the normalised error is given by [18]

$$E_n = \frac{m - m_p}{2 \sqrt{u^2(m) + u^2(m_p)}} , \quad (77)$$

where $u(m)$ is the uncertainty evaluated by the software and $u(m_p)$ is the uncertainty provided by the User (worksheet “Data”). The calculation does not consider the correlation between the two measurements.

Appendix B : Error Messages

This section provides a list of messages that may be displayed during software execution and their description:

Generic

The input data file must be an Excel workbook. *Input files must be Excel files, message appears if non-Excel files are selected.*

An input data file must be selected. *The user did not select any input file.*

An output data file must be selected. *The user did not indicate an output file.*

The file is open by Excel, please close the file. *The file that is being saved is already open.*

File Database

The input file Database does not exist, please check the file ini.txt. *The file with the Database does not exist, check the file or correct the ini.txt file.*

The input file Database must contain the worksheets: "Data", "Balances" and "Info". *Database file does not contain the correct worksheets.*

Worksheet "Data"

The worksheet "Data" must contain correct headers. *Incorrect column headers for worksheet "Data".*

Some input data in worksheet 'Data' for weight *Name Weight* are not numeric. *Non numerical data.*

Some input data in worksheet 'Data' for weight *Name Weight* are negative. *Negative data.*

Some nominal values of the weights are missing. *Missing data.*

Some Mass values of the weights are missing. *Missing data.*

Some uncertainty values of the weights are missing. *Missing data.*

Volume or density for the weights must be provided, check worksheet "Data". *Missing data.*

Some uncertainties for Volume are missing, check worksheet "Data". *Missing data.*

Some uncertainty for Density are missing, check worksheet "Data". *Missing data.*

Some names composed of Set+ID have a length of more than 65 characters. *The name composition Set+ID has to be a length of no more than 65 characters.*

Worksheet “Balances”

The worksheet "Balances" must contain correct headers. *Incorrect column headers for worksheet "Balances".*

Check worksheet "Balances" some information on balances is missing. *Missing data.*

Check worksheet “Balances”, for the balance *NameBalance* some information on correction positions is wrong. Correct format is number_number and then the cell with the correction value. *Incorrect information for position correction.*

Check worksheet “Balances”, for the balance *NameBalance* some correction positions are equal. *Position correction data are repeated several times.*

Check worksheet “Balances”, for the balance *NameBalance* some correction positions are redundant y_x and x_y . *Position corrections provided in the form y_x and x_y .*

Some value for Adjustment mass or Reading Adjustment mass are not greater than zero. *Negative or null values for the adjustment mass or the corresponding reading.*

Some value for Density Adj mass out of the range from 1000 to 30000 kg/m³. *Values out of range.*

Balance *NameBalance* found n times in file Data Base. *Balance entered several times in the database file.*

There is probably a mistake on the values of the Adjustment weight or the associated reading of the balance, some differences are $> 0.1\%$. *Values out of range.*

Worksheet “Info”

The worksheet "Info" must contain correct headers. *Incorrect column headers for worksheet "Info".*

Check worksheet "Info" must contain values for: $u(\text{air dens})$, air dens correl , $\text{references standards correlation}$, g gradient, $u(g$ gradient), $u(h)$. *Missing data.*

Check cell "hr%/tdw" worksheet "Info". *If the air density is calculated using environmental parameters, please indicate how the humidity is measured.*

Worksheet “WP”

The input data file must contain the worksheet "WP". *Missing worksheet "WP".*

The worksheet "WP" must contain correct headers. *Incorrect column headers for worksheet "WP".*

Weight *NameWeight* found n times in file Data Base. *Weight entered several times in the database file*

NameWeight no found, check files. *The standard on the "WP" worksheet is not in the database.*

Some nominal values of the weights are missing. *Missing data.*

In the sheet "WP" the definitions of the type of weights: Reference, Check and Test are not correct. Check the entries, at least one standard has to be a reference. *Wrong data.*

Worksheet "Corr"

Matrix correlation for reference standards is present, but not correct. It will be use the value = x.x. *There are mistakes in the matrix for the references correlation. The value of the sheet "Info" will be used.*

Worksheet "WD"

The input data file must contain the worksheet "WD". *Missing worksheet "WD".*

The worksheet "WD" must contain correct headers. *Incorrect column headers for worksheet "WD".*

The matrix "WD" must contain only 0, -1, 1. *Invalid data.*

The number of column of matrix "WD" must be equal to the number of the weights in worksheet "WP". *Wrong WD matrix size.*

The number of rows of matrix are not enough, it is necessary to use more than one reference standard. *The system of equations can only be solved by using more than one reference standard.*

Check input data of the weighing design, some differences are not zero. *Some weighing differences are not null.*

Invalid Equation. *In the construction of the equation from the front panel, the difference of the nominal values is not zero.*

Worksheet "Pos"

Check matrix dimension worksheet "Pos", not consistent with matrix in worksheet "WD". *Wrong Pos matrix size.*

Check the entries of the worksheet "Pos". *Invalid data, letters can be 'z' or capital letters.*

Check matrix "Pos", the position of some weights is not correct. *The position of some weights is wrong.*

Check matrix "Pos", at least one position for the weights group A and B has to be "z", or "z" is in a wrong column position. *At least one position for group has to be "z".*

Check matrix "Pos", some weights are placed in wrong position, not in accord with the design matrix. *The positions of the weights have to be consistent with the design matrix.*

Check matrix dimension worksheet "Pos", Maximum number of weights is 100. *For the matrix "Pos" the maximum number of weights is 100.*

Worksheet "Meas"

The input data file must contain the worksheet "Meas". *Missing worksheet " Meas ".*

The worksheet "Meas" must contain correct headers. *Incorrect column headers for worksheet "Meas".*

The number of equations is not consistent with the number of rows of the design matrix. *Wrong number of equations in the 'Meas' worksheet.*

Some mass differences or uncertainties u_{Diff} in worksheet "Meas" are missing. *Missing data.*

Some air temperature in worksheet "Meas" are missing, or the range is outside from 10 °C to 30 °C. *Values out of range.*

Some humidity values out from the range 0 to 100. *Values out of range.*

Some pressure values out from the range 60000 Pa to 105000 Pa. *Values out of range.*

Some CO₂ values out from the range 100 ppm to 1000 ppm. *Values out of range.*

As air density is not provided it is necessary to provide: humidity, pressure and CO₂. *Missing data.*

NameBalance no found, check files. *The balance in the worksheet "Meas" is not in the database.*

Check worksheet "Meas", position x_y of the balance *NameBalance* no found, correction will be zero. *Correction for the position not found.*

Calibration Parameters

Value for air density uncertainty not numeric or negative, the value is restored to its previous value. *Value out of range.*

Value for reference standards correlation not numeric, negative, or >1 , the value is restored to its previous value. *Value out of range.*

Value for correlation air density not numeric, negative or >1 , the value is restored to its previous value. *Values out of range.*

Value for gradient of g not numeric or negative, the value is restored to its previous value. *Value out of range.*

Value for uncertainty of gradient of g not numeric or negative, the value is restored to its previous value. *Value out of range.*

Value for uncertainty of height of the weights not numeric or negative, the value is restored to its previous value. *Value out of range.*

Elaboration

Only one has to be the reference. *In Lagrange multipliers method there can only be one reference.*

Not applicable, number of equations or number of reference standards not enough. *Problem that cannot be solved (Gauss-Markov).*

Not applicable, number of equations not enough. *Problem that cannot be solved (Lagrange multipliers).*

Table Results

Type can be: "R" for Reference; "C" for Control; "T" for under Test. *Wrong data.*

At least one weight must be a reference. *No weight is selected as reference.*

Record historical data

Please select at least one entry on the column Save of the table Results. *No weight is selected.*

Windows Select Weights

Weight already insert. *The selected weight is already in the list.*

Windows Built Scheme

Invalid Equation, all inputs are null. *Invalid data.*

The inputs must contain only 0, -1, 1. *Invalid data.*

Invalid equation the sum is not zero. *Invalid data.*

The number of rows of matrix are not enough, it is necessary to use more than one reference standard. *For the solution more than one reference standard has to be use.*

Appendix C : Examples

The examples presented in this Appendix demonstrate in different ways how the instructions contained in this guideline may be applied correctly. They are not intended to indicate any preference for certain procedures as against others for which no example is presented.

Where a calibration laboratory wishes to proceed in full conformity to one of the examples, it may make reference to it in its quality manual.

Examples C1 provides an approach for the mass scale calibration by the subdivision method for a decade from 100 g to 10 g. Example C2 provides a robust approach for calibration of mass standards of the same nominal value. Example C3 provides a method for calibration of mass standards by the one-to-one method. The characteristics of the mass standards used in the examples are shown in table C1 (worksheet “Data”). The database is composed of three sets of weights, there are reference standards, check standards and weights of unknown value. The characteristics of the balances are shown in table C2. For the balance “M 1kg” the sensitivity is $S=1$, and two corrections for the positions are given, for the balance “MM” the reading for the balance will be corrected for the sensitivity. The calibration parameters are shown in table C3. The file with this information is “DB example.xlsx”.

Set of We	ID	Nom [g]	Value [g]	u (k=1) [g]	Volume [u(k=1) [cm]	Density [k u(k=1) [kg]	HCoG [cm]	h [cm]	Alfa x10 ⁻⁶	u(k=1) [1/		
Nim	100 Nim	100	99,9999650	0,000003	12,5122	0,00013		0,4	0,8	36,6	2	
Nim	100 Nim*	100	100,0001587	0,000003	12,5122	0,00013		0,4	0,8	36,6	2	
H15	50 h*	50	50,0000000	1	6,2418	0,00015		0,8	1,6	45	2	
H15	50 h**	50	50,0000000	1	6,2423	0,00020		0,8	1,6	45	2	
Nim	20**	20	20,0000000	1	2,5019	0,0001327		0,25	0,5	36,6	2	
Nim	20***	20	20,0000234	0,000002	2,502	0,0001327		0,25	0,5	36,6	2	
Nim	10	10	10,0000000	1	1,2515	0,00018		0,125	0,25	36,6	2	
Nim	10*	10	10,0000000	1	1,2511	0,00018		0,125	0,25	36,6	2	
INRIM	1 kg 62	1000	999,999104	0,000023	46,435	0,0003		1,95	3,9	25,982	2	
INRIM	1 kg H	1000	999,999277	0,000026	124,2306	0,0003		3,5	7,5	45	2,5	
INRIM	1 kg* H	1000	999,999134	0,000026	124,2319	0,00048		3,5	7,5	45	2,5	
INRIM	1 kg M 19	1000	1000,000000	1			8000	8	3,5	7,5	45	2,5
INRIM 2	1 kg E2	1000	1000,0002	0,00095	125,7862	0,7911081						
INRIM 2	1kg T	1000	1000	1			8000	100				
INRIM 2	500 g	500	500,000012	0,00005	62,522	0,005						
INRIM 2	500 g T	500	500	1			8000	100				

Table C1. Worksheet “Data” used for the examples

Model	Capacity [Resolutio	Adj mass [u adj mass	Dens Adj	u ecc [mg]	u lin [mg]	Read Adj	r pos	corr [m	pos	corr [mg]
M 1kg	1000	0,001	10	0,005	8000	0	0	10	1_2	0,0015	2_3	-0,0015
MM	1000	0,0001	1,00001	0,001	8000	0	0	1,000075				

Table C2. Worksheet “Balances” used for the examples

u air dens	air dens c	reference	g gradient	u (g gradi	u(h) [cm]	hr%/tdw
0,0005	0,5	0,95	0,0024	0,0001	0,01	tdw

Table C3. Worksheet “Info” used for the examples

Example C1 : Calibration of a decade from 100 g to 10 g

The weights selected for this calibration are shown in table C4 (worksheet “WP”),

Set	ID	Nom.Value/g
Nim	100 Nim	100
Nim	100 Nim*	100
H15	50 h*	50
H15	50 h**	50
Nim	20**	20
Nim	20***	20
Nim	10	10
Nim	10*	10

Table C4. Weighing Plan for the example C1

The weights of 100 g are calibrated mass standards, which can be used as reference or check standards. An additional standard whose value is known is the “20***”, which will be used as a check standard. The weighing design consists of 10 different comparisons, in Table C5 the design matrix is shown, the matrix of 1,-1 and 0 is the worksheet “WD”.

100 Nim	100 Nim*	50 h*	50 h**	20**	20***	10	10*
1	-1	0	0	0	0	0	0
1	0	-1	-1	0	0	0	0
0	1	-1	-1	0	0	0	0
0	0	1	-1	0	0	0	0
0	0	1	0	-1	-1	-1	0
0	0	0	1	-1	-1	0	-1
0	0	0	0	1	-1	0	0
0	0	0	0	1	0	-1	-1
0	0	0	0	0	1	-1	-1
0	0	0	0	0	0	1	-1

Table C5. Weighing Design for the example C1

The measurements are performed with the “MM” balance, a comparator with resolution of 0,1 µg. Each comparison is repeated five times, the standard deviation of the mass differences are within 0,2 µg. The input uncertainty contribution of the weighing (equal for all measurements) is 0,000 2 mg (“u Diff mg”) which takes into account the standard deviation of the mean value of the weighing differences and also the instability of the weights during the measurements. The measurements are shown in Table C6, that is the worksheet “Meas”.

Row	Diff. Balance mg	Temp. °C	hr%/tdw	Patm Pa	CO2 ppm	rho air kg/m3	u Diff mg	Balance	Positions	Date	Name	Note
1	-0,19169	19,32				1,18575	0,0002	MM				
2	-0,04163	19,35				1,15080	0,0002	MM				
3	0,14972	19,35				1,15080	0,0002	MM				
4	-0,00282	19,37				1,16842	0,0002	MM				
5	-0,02325	19,31				1,15676	0,0002	MM				
6	-0,06409	19,39				1,16841	0,0002	MM				
7	-0,00285	19,46				1,15520	0,0002	MM				
8	0,02439	19,47				1,15518	0,0002	MM				
9	0,02718	19,44				1,15524	0,0002	MM				
10	-0,04330	19,38				1,15290	0,0002	MM				

Table C6. Measurements for the example C1

The grouping method is given by the worksheet “Pos”, where some groups of weights are stacked and also side-by-side (Fig. C1).

	A	B	C	D	E	F	G	H
1	z	z	0	0	0	0	0	0
2	z	0z	z	z	0	0	0	0
3	0z	z	z	z	0	0	0	0
4	0	0z	z	z	0	0	0	0
5	0	0z	0z	0z	E	F		0
6	0	0	0z	z	E		0F	
7	0	0	0	0z	z		0	0
8	0	0	0	0z		0z	G	
9	0	0	0	0	0z	z	G	
10	0	0	0	0	0	0z	z	
11								
12								

Figure C1. Grouping scheme for the example C1

- **Evaluation by Lagrange multipliers**

The reference standard selected is the “100 Nim”, the “100 Nim*” and the “Nim 20***” are used as check standards, the settings are shown in Fig. C2

Results							
	Type	Value/g	m-m _N /mg	u/mg (k=1)	Δ m/mg	U(Δ m) /mg	En
Nim 100 Nim	R						
Nim 100 Nim*	C						
H15 50 h*	T						
H15 50 h**	T						
Nim 20**	T						
Nim 20***	C						
Nim 10	T						
Nim 10*	T						

Figure C2. Settings of the mass standards

In the table C7 the results are shown. Worksheet “Results”.

Set	ID	Type	Value/g	m-m _N /mg	u/mg (k=1)	Delta m/mg	U(Delta m)/mg (k=2)	En	sigma/mg
Nim	100 Nim	R	99.9999650	-0.0350	0.0030	0.0000	0.0085	0.000	0.00019
Nim	100 Nim*	C	100.0001566	0.1566	0.0030	-0.0021	0.0085	-0.252	
H15	50 h*	T	49.9999856	-0.0144	0.0015				
H15	50 h**	T	49.9999889	-0.0111	0.0015				
Nim	20**	T	20.0000220	0.0220	0.0006				
Nim	20***	C	20.0000250	0.0250	0.0006	0.0015	0.0042	0.363	
Nim	10	T	9.9999777	-0.0223	0.0004				
Nim	10*	T	10.0000207	0.0207	0.0004				

Table C7. Results for example C1 by Lagrange

The validation of the results is performed by checking the obtained residuals (Table C8) and the results of the two check standards. For these measurements the residuals are within the resolution of the balance and also the results of the check standards are consistent. For the “100 g Nim*” the deviation is - 0,0021 mg. In addition, the standard uncertainty of the fit is 0,00019 which is similar to the typical standard deviation of the balance.

Res/mg	dev_lim/mg	Test
-0.0001	0.0004	true
0.0001	0.0004	true
-0.0001	0.0004	true
-0.0001	0.0004	true
0.0001	0.0004	true
-0.0001	0.0004	true
-0.0000	0.0004	true
0.0000	0.0004	true
-0.0000	0.0004	true
0.0001	0.0004	true

Table C8. Residuals for example C1 by Lagrange

- Evaluation by Gauss-Markov

With the Gauss-Markov method and the same settings of the Lagrange method the results are identical. By using the Gauss-Markov method, it is possible to use as reference standards, the two weights of 100 g, the setting is shown in Fig. C3. In this case the correlation between the two references is used, the input value is 0,95.

Results							
	Type	Value/g	m-m _N /mg	u/mg (k=1)	Δ m/mg	U(Δ m)/mg	En
Nim 100 Nim	R						
Nim 100 Nim*	R						
H15 50 h*	T						
H15 50 h**	T						
Nim 20**	T						
Nim 20***	C						
Nim 10	T						
Nim 10*	T						

Figure C3. Settings of the mass standards with two references

In table C9 the results obtained with this setting are shown. It can be observed that the results are slightly different from those obtained with a single reference.

The two reference standards have the same uncertainty, however the mass difference of the two references with respect the values in the database is 0,0021 mg (see C1 Lagrange example), for this reason, when using the two references, the values are adjusted to compensate this difference, in fact, it can be seen that the differences (“Delta m/mg”) for the two references is 0.001 mg (positive for one and negative for the other). In practice, since the uncertainties of the two reference standards are identical, the result obtained is equivalent to the average value between the values obtained by performing the two calibrations separately with the two references.

When using several references, it is essential to check the consistency of the new values of the reference standards, which must be consistent with the values provided from the database file, which are the reference values. Since the software for the E_n calculation does not take into account the correlation between the previously set value and the newly calculated value, it is a good rule to check that the variation is within the expanded uncertainty. The validation of the results with a value of E_n close to 1 may not be sufficient. If the results are not consistent, it is a sign of some mistake or instability.

Set	ID	Type	Value/g	m-m _N /mg	u/mg (k=1)	Delta m/mg	U(Delta m)/mg (k=2)	En	sigma/mg
Nim	100 Nim	R	99.9999660	-0.0340	0.0030	0.0010	0.0084	0.118	0.00073
Nim	100 Nim*	R	100.0001577	0.1577	0.0030	-0.0010	0.0084	-0.118	
H15	50 h*	T	49.9999861	-0.0139	0.0015				
H15	50 h**	T	49.9999894	-0.0106	0.0015				
Nim	20**	T	20.0000222	0.0222	0.0006				
Nim	20***	C	20.0000252	0.0252	0.0006	0.0018	0.0042	0.422	
Nim	10	T	9.9999779	-0.0221	0.0004				
Nim	10*	T	10.0000208	0.0208	0.0004				

Table C9. Results for example C1 by Gauss-Markov and two references

In table C10 the residuals for this example are shown.

Res/mg	dev_lim/mg
0.0000	0.0004
0.0002	0.0004
-0.0002	0.0004
-0.0001	0.0004
0.0001	0.0004
-0.0001	0.0004
-0.0000	0.0004
0.0000	0.0004
-0.0000	0.0004
0.0001	0.0004

Table C10. Residuals for example C1 by Gauss-Markov and two references

Although the residuals and standard deviation of the fit are higher than in the case of a single reference, the result obtained could be considered more robust as it is based on two reference standards.

The following tables show the values of the measurements, worksheet “Measurements” and the input uncertainties, worksheet “Input uncertainties” of the output file.

Diff Bal/mg	Corr.pos./mg	Sens	Delta V/cm ³	Rho_a/kg/m ³	Corr Buoy/mg	hgA/cm	hgB/cm	Delta_h/cm	Corr g/mg	Dm/mg
-0,1917	0	0,999787	0,0000	1,1858	0,0000	0,4	0,4	0,000	0,0000	-0,1917
-0,0416	0	0,999791	0,0282	1,1508	0,0324	0,4	0,8	-0,40	-0,0001	-0,0093
0,1497	0	0,999791	0,0282	1,1508	0,0324	0,4	0,8	-0,40	-0,0001	0,1820
-0,0028	0	0,999789	-0,0005	1,1684	-0,0006	0,8	0,8	0,000	0,0000	-0,0034
-0,0233	0	0,999791	-0,0136	1,1568	-0,0158	0,8	0,625	0,175	0,0000	-0,0390
-0,0641	0	0,999789	-0,0127	1,1684	-0,0149	0,8	0,625	0,175	0,0000	-0,0789
-0,0029	0	0,999791	-0,0001	1,1552	-0,0001	0,25	0,25	0,000	0,0000	-0,0030
0,0244	0	0,999791	-0,0007	1,1552	-0,0008	0,25	0,25	0,000	0,0000	0,0236
0,0272	0	0,999791	-0,0006	1,1552	-0,0007	0,25	0,25	0,000	0,0000	0,0265
-0,0433	0	0,999791	0,0004	1,1529	0,0005	0,125	0,125	0,000	0,0000	-0,0428

Table C11. Measurements data for the C1 example

Res.Bal./mg	Sens./mg	Ecc./mg	Lin./mg	CoG/mg	m_Diff./mg	c_V/mg	c_air/mg	Add./mg
4,08E-05	1,92E-07	0,00E+00	0,00E+00	3,39E-06	2,00E-04	2,20E-04	0,00E+00	0,00E+00
4,08E-05	4,16E-08	0,00E+00	0,00E+00	1,25E-05	2,00E-04	3,25E-04	1,41E-05	0,00E+00
4,08E-05	1,50E-07	0,00E+00	0,00E+00	1,25E-05	2,00E-04	3,25E-04	1,41E-05	0,00E+00
4,08E-05	2,82E-09	0,00E+00	0,00E+00	1,70E-06	2,00E-04	2,92E-04	2,50E-07	0,00E+00
4,08E-05	2,33E-08	0,00E+00	0,00E+00	2,18E-06	2,00E-04	3,47E-04	6,82E-06	0,00E+00
4,08E-05	6,41E-08	0,00E+00	0,00E+00	2,18E-06	2,00E-04	3,83E-04	6,37E-06	0,00E+00
4,08E-05	2,85E-09	0,00E+00	0,00E+00	6,79E-07	2,00E-04	2,17E-04	5,00E-08	0,00E+00
4,08E-05	2,44E-08	0,00E+00	0,00E+00	6,79E-07	2,00E-04	3,32E-04	3,50E-07	0,00E+00
4,08E-05	2,72E-08	0,00E+00	0,00E+00	6,79E-07	2,00E-04	3,32E-04	3,00E-07	0,00E+00
4,08E-05	4,33E-08	0,00E+00	0,00E+00	3,39E-07	2,00E-04	2,93E-04	2,00E-07	0,00E+00

Table C12. Input uncertainties for the C1 example

Example C2 : Calibration of mass standards of the same nominal value with a robust design matrix

The weights selected for this calibration are shown in Table C13 (worksheet “WP”).

Set	ID	Nom.Value/g
INRIM	1 kg 62	1000
INRIM	1 kg H	1000
INRIM	1 kg* H	1000
INRIM	1 kg M 19	1000

Table C13. Weighing Plan for the example C2

The weight “INRIM 1 kg 62” is a Pt-Ir mass standard, which can be used as reference. Two additional standards whose value is known are the INRIM “1 kg H” and “1 kg* H”. For the mass standard of “1 kg M 19” only the density is provided.

The weighing design consists of 12 different comparisons, in Table C14 the design matrix is shown

1 kg 62 (pos1)	1 kg H (pos2)	1 kg* H (pos3)	1 kg M 19 (pos4)
1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	1	0	-1
0	0	1	-1
-1	1	0	0
-1	0	1	0
-1	0	0	1
0	-1	1	0
0	-1	0	1
0	0	-1	1

Table C14. Weighing Design for the example C2

The measurements are performed with the “M1kg” balance, a comparator with four positions and resolution of 1 µg. Each comparison is repeated for five times, the standard deviation of the mass differences are within 0.7 µg.

The weighing uncertainty contribution “u Diff “ is 0,0005 mg which takes into account the standard deviation of the mean value of the weighing differences and also the instability of the weights during the measurements. This balance is affected by position errors for the positions 1 to 2 and 2 to 3, the corrections are taken into account (see Table C16).

The “Corr” worksheet defining the correlation between the three standards was also used, in Table C15 the values are shown.

	1 kg 62	1 kg Hae	1 kg* Hae	1 kg Me 19
1 kg 62	1	0,9	0,9	
1 kg Hae		1	0,95	
1 kg* Hae			1	
1 kg Me 19				1

Table 15. Correlations between reference standards

For this example, the air density uncertainty is 0,0001 kg/m³, this value is set by the GUI.

The measurements are shown in Table C16.

Row	Diff. Balance mg	Temp. °C	hr %/tdw	Patm Pa	CO2 ppm	rho air kg/m3	u Diff mg	Balance	Positions
1	91,2804	19,06	8,51	98977	523	1,17538	0,0005	M 1kg	1_2
2	91,4177	19,06	8,51	98977	523	1,17538	0,0005	M 1kg	
3	90,2999	19,06	8,51	98976	523	1,17537	0,0005	M 1kg	
4	0,1375	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	2_3
5	-0,9799	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	
6	-1,1176	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	
7	-91,2797	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	2_1
8	-91,4177	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	
9	-90,2996	19,06	8,50	98976	523	1,17537	0,0005	M 1kg	
10	-0,1368	19,06	8,49	98976	523	1,17537	0,0005	M 1kg	3_2
11	0,9801	19,06	8,49	98976	523	1,17536	0,0005	M 1kg	
12	1,1178	19,06	8,49	98976	523	1,17536	0,0005	M 1kg	

Table C16. Measurements for the example C2

In this case, being a one-to-one comparison, there is no difference on the selection of the grouping method.

Evaluation by Gauss-Markov

The reference standard selected is the “1 kg 62”, the values of the weights “1 kg H” and “1 kg H*” are known from the previous calibration, to check the variation obtained with the new calibration, they are set as check standards, the settings are shown in Fig. C4.

Results							
	Type	Value/g	$m-m_N$ /mg	u /mg (k=1)	Δ m/mg	$U(\Delta m)$ /mg	En
INRIM 1 kg 62	R						
INRIM 1 kg H	C						
INRIM 1 kg* H	C						
INRIM 1 kg M 19	T						

Figure C4. Settings of the mass standards

In the table C17 the results are shown.

Set	ID	Type	Value/g	$m-m_N$ /mg	u /mg (k=1)	Delta m/mg	$U(\Delta m)$ /mg (k=2)	En	sigma/mg
INRIM	1 kg 62	R	999.9991040	-0.8960	0.0230	0.0000	0.0651	0.000	0.00067
INRIM	1 kg H	C	999.9992741	-0.7259	0.0238	-0.0029	0.0705	-0.041	
INRIM	1 kg* H	C	999.9991393	-0.8607	0.0238	0.0053	0.0705	0.075	
INRIM	1 kg M 19	T	1000.0011590	1.1590	0.1488				

Table C17. Results for example C2 by Gauss-Markov

The validation of the results is performed checking the obtained residuals (Table C18) and from the variation with respect the previous values of the weights “1 kg H” and 1 kg* H”. For these measurements the residuals are within the limit evaluated by the resolution of the balance and the weighing uncertainty.

Res/mg	dev_lim/mg	Test
0.0003	0.0013	true
-0.0002	0.0013	true
-0.0004	0.0013	true
-0.0003	0.0013	true
0.0009	0.0013	true
-0.0005	0.0013	true
-0.0004	0.0013	true
-0.0005	0.0013	true
0.0004	0.0013	true
0.0011	0.0013	true
-0.0006	0.0013	true
0.0007	0.0013	true

Table C18. Residuals for example C2

The following tables show the values of the measurements, worksheet “Measurements” and the input uncertainties, worksheet “Input uncertainties” of the output file.

Diff Bal/mg	Corr.pos./mg	Sens	Delta V/cm ³	Rho_a/kg/m ³	Corr Buoy/mg	hgA/cm	hgB/cm	Delta_h/cm	Corr g/mg	Dm/mg
91,2804	0,0015	0,999853	-77,792	1,17538	-91,4346	1,95	3,5	-1,55	-0,0037	-0,1698
91,4177	0	0,999853	-77,793	1,17538	-91,4360	1,95	3,5	-1,55	-0,0037	-0,0355
90,2999	0	0,999853	-78,561	1,17537	-92,3384	1,95	3,5	-1,55	-0,0037	-2,0554
0,1375	-0,0015	0,999853	-0,001	1,17537	-0,0015	3,5	3,5	0	0,0000	0,1345
-0,9799	0	0,999853	-0,769	1,17537	-0,9043	3,5	3,5	0	0,0000	-1,8840
-1,1176	0	0,999853	-0,768	1,17537	-0,9028	3,5	3,5	0	0,0000	-2,0202
-91,2797	-0,0015	0,999853	77,792	1,17537	91,4338	3,5	1,95	1,55	0,0037	0,1697
-91,4177	0	0,999853	77,793	1,17537	91,4354	3,5	1,95	1,55	0,0037	0,0348
-90,2996	0	0,999853	78,561	1,17537	92,3380	3,5	1,95	1,55	0,0037	2,0554
-0,1368	0,0015	0,999853	0,001	1,17537	0,0015	3,5	3,5	0	0,0000	-0,1337
0,9801	0	0,999853	0,769	1,17536	0,9043	3,5	3,5	0	0,0000	1,8842
1,1178	0	0,999853	0,768	1,17536	0,9027	3,5	3,5	0	0,0000	2,0204

Table C19. Measurements data for the C2 example

Res.Bal./mg	Sens./mg	Ecc./mg	Lin./mg	CoG/mg	m_Diff./mg	c_V/mg	c_air/mg	Add./mg
0,00041	0,00005	0	0	0,00016	0,0005	0,0006	0,0078	0
0,00041	0,00005	0	0	0,00016	0,0005	0,0008	0,0078	0
0,00041	0,00005	0	0	0,00016	0,0005	0,1469	0,0079	0
0,00041	0,00000	0	0	0,00003	0,0005	0,0008	0,0000	0
0,00041	0,00000	0	0	0,00003	0,0005	0,1469	0,0001	0
0,00041	0,00000	0	0	0,00003	0,0005	0,1469	0,0001	0
0,00041	0,00005	0	0	0,00016	0,0005	0,0006	0,0078	0
0,00041	0,00005	0	0	0,00016	0,0005	0,0008	0,0078	0
0,00041	0,00005	0	0	0,00016	0,0005	0,1469	0,0079	0
0,00041	0,00000	0	0	0,00003	0,0005	0,0008	0,0000	0
0,00041	0,00000	0	0	0,00003	0,0005	0,1469	0,0001	0
0,00041	0,00000	0	0	0,00003	0,0005	0,1469	0,0001	0

Table C20. Input uncertainties for the C2 example

The calibration has also been performed considering the weights “1 kg 62” and “1 kg H” as references, the results are shown in table C21. Since the uncertainty of the “1 kg 62” weight is smaller than that of the “1 kg H” weight, the values are very similar to that obtained with only “1 kg 62” as a reference.

Set	ID	Type	Value/g	m-m_N/mg	u/mg (k=1)	Delta m/mg	U(Delta m)/mg (k=2)	En	sigma/mg
INRIM	1 kg 62	R	999.9991042	-0.8958	0.0230	0.0002	0.0650	0.003	0.00102
INRIM	1 kg H	R	999.9992748	-0.7252	0.0235	-0.0022	0.0701	-0.031	
INRIM	1 kg* H	C	999.9991400	-0.8600	0.0235	0.0060	0.0701	0.085	
INRIM	1 kg M 19	T	1000.0011597	1.1597	0.1488				

Table C21. Results for example C2 by Gauss-Markov and two references

Example C3 : Calibration of mass standards by the one-to-one method.

This example shows a calibration one-to-one, with two references of different nominal mass. The weights selected for this calibration are shown in table C22 (worksheet “WP”).

Set	ID	Nom.Value/g	
INRIM 2	1 kg E2	1000	R
INRIM 2	1kg T	1000	T
INRIM 2	500 g	500	R
INRIM 2	500 g T	500	T

Table C22. Weighing Plan for the example C3

In this example the “Type” of the mass standards are defined by the worksheet “WP” , as shown in Table C22.

For this example, the mass standard “1kg E2” is a OIML weight of Class E2, whose density was approximately estimated, it is used as reference standard.

The standard “1kg E2” was calibrated in conventional mass, in table C23 the value m_c is shown

m_c/g	$u(m_c)/g$
999,999 256 5	0,000 248

Table C23. Reference value in conventional mass for the “1kg E2” weight

As the RealMass Calibration software uses the mass values in term of “true” mass, it is necessary to convert the value from conventional mass to “true” mass. For the uncertainty conversion, the value of the density (or volume) and also of the correlation term between mass and volume, or the air density at the time of the calibration, are required.

Considering that the density is $7,850 \text{ g/cm}^3$ and the standard uncertainty is $0,050 \text{ g/cm}^3$, the estimated volume is $V=125,7862 \text{ cm}^3$ and $u(V) = 0.7911 \text{ cm}^3$, the air density at the time of the calibration was $1,160 \text{ kg/m}^3$. The value is converted in mass by equation (6), and the uncertainty by equation (8), in table C24 the values are shown, these values are reported in the database file, worksheet “Data”.

m/g	$u(m)/g$
1000,000 200	0,000950

Table C24. Reference value in mass for the standard 1kg E2

As the density uncertainty is not negligible, in order to provide the completed information for the calibration, it is necessary to provide the correlation $r(m,V)$, which is evaluated by equations (3) and (4), the value is $r(m,V) = 0,965985$, which is entered in the worksheet “Data”.

The weighing design consists of two comparisons, in Table C25 the design matrix is shown.

1 kg E2	1kg T	500 g	500 g T
1	-1	0	0
0	0	1	-1

Table C25. Weighing Design for the example C3

In table C26 the measurements are shown.

Row	Diff. Balance mg	Temp. °C	hr %/tdw	Patm Pa	CO2 ppm	rho air kg/m3	u Diff mg	Balance
1	0,000000	20,0				1,160	0,0005	M 1kg
2	0,000530	20,1				1,160	0,0005	M 1kg

Table C26. Measurements for the example C3

Since two references are specified, it is necessary to use the Gauss-Markov method. In table C27 the results in terms of mass, worksheet “Results” are shown. The result in term of conventional mass are shown in Table C28.

Set	ID	Type	Value/g	m-m _N /mg	u/mg (k=1)	Delta m/mg	U(Delta m)/mg (k=2)	En
INRIM 2	1 kg E2	R	1000.0002000	0.2000	0.9500	0.0000	2.6870	0.000
INRIM 2	1kg T	T	999.999289	-0.712	1.829			
INRIM 2	500 g	R	500.0000120	0.0120	0.0500	0.0000	0.1414	0.000
INRIM 2	500 g T	T	499.9999860	-0.0140	0.9076			

Table C27. Results in terms of mass for example C3 by Gauss-Markov

Set	ID	Type	Value/g	m-m _N /mg	u/mg (k=1)
INRIM 2	1 kg E2	R	999.9992565	-0.7435	0.2477
INRIM 2	1kg T	T	999.9992879	-0.7121	0.2535
INRIM 2	500 g	R	499.9999856	-0.0144	0.0497
INRIM 2	500 g T	T	499.9999859	-0.0141	0.0593

Table C28. Results in terms of conventional mass for example C3 by Gauss-Markov

The following tables show the values of the measurements, worksheet “Measurements” and the input uncertainties, worksheet “Input uncertainties” of the output file.

Diff Bal/ mg	Corr.pos./ mg	Sens	Delta V/ cm ³	Rho_a/ kg/m ³	Corr Buoy/ mg	hgA/ cm	hgB/ cm	Delta_h/ cm	Corr g/ mg	Dm/ mg
0,00000	0	0,999855	0,7862	1,160	0,9120	0	0	0	0	0,9120
0,00053	0	0,999855	0,0220	1,160	0,0255	0	0	0	0	0,0260

Table C29. Measurements data for the C3 example

Res.Bal./ mg	Sens./ mg	Ecc./ mg	Lin./ mg	CoG/ mg	m_Diff./ mg	c_V/ mg	c_air/ mg	Add./ mg
0,0004	0	0	0	0	0,0005	2,0316	0,0004	0
0,0004	0	0	0	0	0,0005	0,9063	0,0000	0

Table C30. Input uncertainties for the C3 example